A Partisan Solution to Partisan Gerrymandering: The Define-Combine Procedure*

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Abstract

Redistricting reformers have proposed many solutions to the problem of partisan gerrymandering, including non-partisan commissions and bipartisan commissions with members from each party. Redistricting litigation frequently ends with court- or special-master-drawn plans. All of these methods require either bipartisan consensus or the agreement of both parties on the legitimacy of a neutral third party to resolve disputes. In this paper we propose a new method for drawing districting maps, the Define-Combine procedure, that substantially reduces partisan gerrymandering without requiring a neutral third party or bipartisan agreement. First, one party defines a map of 2N equal-population contiguous districts. Then the second party combines pairs of contiguous districts to create the final map. We use simulations and map-drawing algorithms to show that this procedure reduces partisan bias and produces maps with similar results to those drawn by independent commissions without requiring cooperation or non-partisan actors.

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1 Introduction

Since the earliest elections in the United States, parties and candidates have accused their opponents of partisan gerrymandering — drawing the lines of legislative districts to benefit a particular party (Griffith 1907; Cox and Katz 2002). Partisan gerrymandering pre-dates the Constitutional Convention and 1st Congress. Griffith (1907) documents at least 16 cases of political gerrymanders that were enacted before the "original" Massachusetts gerrymander of 1812, including cases in Pennsylvania, North Carolina, Virginia, New York, and New Jersey, some of which occurred "about a hundred years before" the term "gerrymandering" became widespread (Martis 2008, p. 120). While the prevalence, degree, and sophistication of partisan gerrymandering has varied over time, it poses a consistent problem for district-based representational systems. Reformers have proposed a plethora of solutions for drawing new district boundaries, including bi-partisan commissions with members from each party, non-partisan geographers, and court- or special-master-drawn maps. All these schemes require a neutral, third-party actor, trusted by both political parties, to choose between plans fairly or to draw a map without bias.[1] However, in today's hyper-partisan environment, there are few independent, third-party actors considered by both sides as able to fulfill this role legitimately. In practice, intense controversies continue to surround redistricting, even in states that have enacted reforms.

In this paper we propose a new method for drawing maps, the *Define-Combine Procedure*, which requires neither a neutral third party nor bipartisan cooperation. We develop a simple framework that allows each party to act in their own partisan self-interest but nonetheless achieves a significantly fairer map than would be drawn by the parties on their own. We divide the districting process into two stages. Suppose a state must be divided into N equally populated districts. First, one party—the "definer"—draws $2N$ contiguous, equal population districts. Then, the second party—the "combiner"—selects contiguous pairs of districts from the set defined by the first party to create the final districts. This method produces N equally populated, contiguous districts. By dividing the responsibility of drawing districts into two separate stages, in which each party retains

[1] The exception is an equally-balanced bipartisan commission, which requires cooperation between the parties to avoid deadlocking. Idaho and Washington serve as examples of states with commissions that have an equal number of members from each party and no independent chair (Idaho) or with a non-voting independent chair (Washington).

complete autonomy in their own stage, the parties counteract each other's partisan ambitions while still maintaining considerable flexibility to achieve other objectives, including maintaining compactness, contiguity, and communities of interest. We show that the Define-Combine procedure tends to produce fairer maps with less partisan bias than when one party draws the district map unilaterally.

The Define-Combine procedure contrasts with currently-utilized solutions to partisan gerrymandering in several ways. First, and perhaps most importantly, it does not require either forging bipartisan support between parties or appointing a third-party arbiter to resolve points of difference. A procedure that sidesteps these common stumbling blocks could reduce the contentious political disputes that accompany decennial redistricting, leading to fairer and more quickly-produced maps. Second, Define-Combine is a process-based solution to partisan gerrymandering; it does not require courts to adjudicate controversial claims of partisan gerrymandering. As a result, state courts could use this approach to resolve redistricting disputes without having to define a standard for identifying partisan gerrymandering in the first place, thereby reducing lengthy and expensive litigation. Third, state legislatures could in theory implement Define-Combine as a process to guide redistricting, either within a legislative committee or the legislature as a whole. For those interested in reforms to the redistricting process, states without a popular initiative process appear unlikely to create independent redistricting commissions, and so other process-based solutions that do not require ceding full control of map drawing will be more likely to appeal to partisans within the legislature.

Our paper proceeds as follows. First, we formally define our redistricting game, and we illustrate the process with a simple example. Second, we characterize an equilibrium concept for the redistricting process that we describe. Employing simulations for both simple hypothetical scenarios and real state maps, we show that each party's optimal strategy is to maximize their partisan self-interest in each stage, and that the equilibrium in each case is a final map that is less biased than the maps drawn by either party on their own. We then use our simulation models to characterize the relationship between the vote distribution in the state and first- or second-mover advantages for each party, and conclude by discussing ways that this procedure could be implemented by legislatures, commissions, and courts.

2 Limitations of Current Partisan Gerrymandering Fixes

We can divide current solutions to partisan gerrymandering into two classes. First, there are solutions that have already been implemented in at least one of the fifty states. These include one-party redistricting (where the majority party in the state legislature unilaterally draws congressional and legislative districts) and redistricting commissions. Second, there are proposals — generally put forth by researchers — that move beyond currently implemented solutions and lay out some other mechanism by which maps are drawn. These include methods where the parties draw districts by alternating back and forth; many of these approaches are inspired by the cake-cutting problem and principles of fair division (i.e., how to divide a good, such as a cake, fairly between two parties).

2.1 Already-Implemented Solutions

Citizens have expressed deep dissatisfaction with redistricting solutions currently adopted in the states. For example, fewer than 25% of respondents in the Cooperative Congressional Election Survey answered affirmatively when asked whether the redistricting process in their state was fair (Schaffner and Ansolabehere 2015), regardless of whether the state used legislature- or commission-based redistricting.[2]

Table 1 reports how each state draws legislative districts, with classifications from Levitt.[3] In total, 32 states[4] draw their U.S House districts maps through the legislature exclusively, while 23 draw their state legislative districts exclusively through the state legislature. When one party acts unilaterally to draw maps, legislature-drawn maps sometimes result in extreme partisan gerrymanders favoring the party in power. Existing remedies to these types of extreme gerrymanders have met with several obstacles.

[2]Though commission-based states did register significantly higher approval rates than legislature-based states.

[3]Justin Levitt, *All About Redistricting*, website: `http://redistricting.lls.edu/who.php`, Accessed 2/16/20, with authors' additional updates.

[4]Five of these 32 states have only have one congressional districts and do not currently actually engage Congressional in redistricting.

Table 1: Redistricting Procedures by State, for U.S. House Districts

Legislature Only (32)		Legislature-Involved Commissions			Independent
		Advisory (7)	Political (2)	Backup (2)	Commissions (7)
Alabama	Nebraska	Iowa	Hawaii	Connecticut	Arizona
Alaska*	Nevada	Maine	New Jersey	Indiana	California
Arkansas	New Hampshire	New York			Idaho
Colorado	New Mexico	Ohio			Michigan
Delaware*	North Carolina	Rhode Island			Montana*
Florida	North Dakota*	Vermont*			Utah[3]
Georgia	Oklahoma[1]	Virginia[2]			Washington
Illinois	Oregon				
Kansas	Pennsylvania				
Kentucky	South Carolina				
Louisiana	South Dakota*				
Maryland	Tennessee				
Massachusetts	Texas				
Minnesota	West Virginia				
Mississippi	Wisconsin				
Missouri	Wyoming*				

Advisory Commission: Assists the legislature in drawing the maps, but the legislature has the ultimate power to approve or alter the final district maps; **Political Commission:** Legislature as a whole isn't officially involved, but the members of the commission are politicians or elected officials; **Backup Commission:** Step in if the legislature does not pass a districting plan by a certain deadline — these vary in their composition and procedures as well, but are almost always comprised of politicians (governor, secretary of state, state legislators, or members selected by political leadership); **Independent Commission:** Commissions that have no politicians or elected officials on them, and whose maps are not subjective to legislature approval. Source: Justin Levitt, *All About Redistricting*, website: `http://redistricting.lls.edu/who.php`, along with authors' updates.

[1] On February 6, 2020, the group People Not Politicians in Oklahoma filed an petition to amend the state constitution to transfer redistricting authority from the legislature to a new independent commission of non-elected officials. (`https://www.sos.ok.gov/documents/questions/810.pdf`)

[2] On February 4, 2020, the Virginia senate passed a proposed amendment to the state constitution that would establish a bipartisan redistricting commission, with a mix of state legislators and citizens chosen by a panel of judges. As of writing, the Virginia house of delegates has yet to vote on the proposal and has offered up a competing measure to establish a non-partisan commission.

[3] For the 2020 redistricting cycle, Utah will have a seven-member independent commission to draw their state legislative and U.S. House districts. While the members cannot be elected officials and are required to be unaffiliated with any political party, they are ultimately appointed by elected officials (Proposition 4, "Independent Advisory Commission on Redistricting Initiative", passed November 6, 2018: `https://elections.utah.gov/Media/Default/2018%20Election/Issues%20on%20the%20Ballot/Proposition%204%20-%20Full%20Text.pdf`).

*State only has one U.S. House district

First, the Supreme Court's recent decision in *Rucho v. Common Cause* (2019) effectively closed the door on federal judicial intervention in partisan gerrymandering litigation. In the case of some extreme gerrymanders, State Supreme Courts have stepped in — recent court decisions have struck down maps in Florida (*League of Women Voters v. Detzner*, 2015), in Pennsylvania (*League of Women Voters v. Commonwealth of Pennsylvania*, 2018), and in North Carolina (*Common Cause v. Rucho*, 2018, reconsidered and reaffirmed 2019).

Second, interventions by State Supreme Courts on a state-by-state basis do not appear to represent a comprehensive long-term solution ensuring fair maps. Unsolved legal and implementation issues continue to present problems for judicial remedies at the state level. In Florida, the courts based their decision on a "Fair Districts" amendment prohibiting partisan gerrymandering, which voters had previously added to the state constitution through a popular initiative.[5] In both the Pennsylvania and the North Carolina rulings, the courts relied on more generic language in the state constitutions ensuring "free elections."[6] However, not all states have constitutions that include language providing bases for legal challenges to gerrymandered district maps. According to the National Conference of State Legislatures (NCSL), just 30 states have some version of "free election" clauses, and only 18 of these also require that elections be either "equal" or "open."[7]

Third, the courts often struggle to effectively adjudicate redistricting litigation. At a minimum, courts must decide (1) how to measure and evaluate partisan gerrymandering,[8] (2) how to compare multiple maps,[9] and (3) at what threshold there is too much gerrymandering. But none of these

[5] Amendment 5: "Legislative districts or districting plans may not be drawn to favor or disfavor an incumbent or political party." https://web.archive.org/web/20101208155829/http://projects.palmbeachpost.com/yourvote/ballot_question/florida/2010/amendment-5-and-6-2010/.

[6] For North Carolina, the courts concluded that the redistricting process was not consistent with a broad reading of Section 10 of the North Carolina State Constitution, which states that "All elections shall be free." Similarly in Pennsylvania, the courts found that the challenged map violated the "Free and Equal Elections" Clause (Article 1, Section 5) of the Pennsylvania State Constitution.

[7] National Conference of State Legislatures, "Free and Equal Election Clauses in State Constitutions" https://www.ncsl.org/research/redistricting/free-equal-election-clauses-in-state-constitutions.aspx

[8] There exists no legal consensus on how to best identify instances of partisan gerrymandering, despite a plethora of new partisan gerrymandering metrics developed in the past few decades. Since the Supreme Court's decision in *Vieth v. Jubelirer* (2004), finding a standard to judge partisan gerrymandering has remained a challenge. Measures like the Efficiency Gap (Stephanopoulos and McGhee 2015), the Mean-Median Difference (McDonald and Best 2015), and Partisan Fairness (King and Browning 1987; Grofman and King 2007) have grown increasingly common, but courts have not settled on one. Each approach has some mix of desirable and undesirable features (Stephanopoulos and McGhee 2018).

[9] Courts sometimes rely on simulated or counterfactual election results in order to create a distribution of possible maps against which the actual or proposed redistricting plans can be compared. Current computational limitations

three issues has been settled. Any solution needs to cut through the "sociological gobbledygook" in a way perceived as non-partisan and legally sound.

Redistricting commissions, used in some form by the remaining states, also do not offer a silver-bullet solution to partisan gerrymandering. Table 1 reports the specific type of commission used in each commission-based state. The redistricting process in the majority of commission-based states is not in fact independent from the state legislature. Advisory commissions assist the legislature as it draws district boundaries, but the legislature approves the maps. In backup-commission states, a commission only plays a role if the legislature fails to pass a districting plan within a certain time period. Political or politician commissions are mainly comprised of elected officials. Finally, independent commissions are distinct from the others as they do not include public officials or legislators.[10]

All told, 11 of the 18 commission-based states do not have a redistricting process for their U.S. House districts meaningfully independent from the state legislature; as a result, the map-drawing process remains subject to the same pressures as in states with legislature-drawn maps. Of the seven states that have now established independent redistricting commissions for redrawing their congressional districts, nearly all rely on a member or members to act neutrally (often these members are not affiliated with either of the two major political parties). The logic behind this design is that the two parties will have to appeal to a neutral arbiter — the independent member(s) of the commission — in order to gain a majority and pass a map. In theory, this could cause both parties to curb their partisan gerrymandering efforts in order to create a fairer map appealing to a

make it impossible to create the full distribution of possible maps, so simulations rely on creating a representative sample of possible maps as a baseline (Cho and Liu 2016). Experts continue to debate whether particular simulation methods create a "true" distribution of possible maps, and the courts must navigate among competing methods, including Cirincione, Darling and O'Rourke (2000), Altman and McDonald (2011), Chen and Rodden (2013) which is later modified in Chen and Cottrell (2016), Cho and Liu (2016), Magleby and Mosesson (2018), Duchin (2018*a*), and Fifield et al. (2019).

[10]Selection methods for independent commissions vary significantly — in Arizona, Idaho, Montana, and Washington majority and minority party leaders appoint commissioners, while judges make appointment decisions in Colorado. Alaska has two members chosen by the governor, two by party leaders in the state legislature, and the last by the state supreme court chief justice. California has a process that involves narrowing a pool of applicants down, randomly selecting some members, and then having those members choose the remaining members. Utah's new independent commission will have all members chosen by state legislators, with the governor choosing the commission chair, though all commission members must not be affiliated with any political party nor have voted in any political party's primary elections in the past five years. In the case of Michigan for the future 2020 redistricting, independent commissions members will be selected randomly from a pool of applicants.

neutral (and presumably more moderate) commission member.[11]

Scholars have not reached a consensus on the benefits of independent commissions. Miller and Grofman (2013) examine the efficacy of redistricting commissions in seven Western states and compare them to five non-commission states in the West. They find that redistricting commissions do not out-perform legislatures when judged by the metric of drawing compact, competitive districts that preserve preexisting political boundaries. (On the other hand, according to Miller and Grofman (2013), commissions excel at producing maps "on time" that avoid litigation.) This research contrasts with Carson and Crespin (2004), who find more competitive districts in commission-drawn maps in the 1990s and 2000s redistricting cycles. Lastly, when Henderson, Hamel and Goldzimer (2018) use simulations to consider a set of alternative maps that could have been enacted by independent commissions, they find independent commissions insulate incumbent legislators to the same degree that party-controlled legislators do, suggesting that independent commissions may not be as neutral as many suppose.

The effectiveness of independent commissions also hinges crucially on who staffs them. A Brennan Center report notes that "the strength and independence of the [commissioner] selection process was, by far, the most important determinant of a commission's success" (*Redistricting Commissions: What Works* 2018). Even with adequate staffing, independent commissions do not quell the partisan anger over redistricting controversies. Miller and Grofman (2013, p. 648) note that "the decisions of such commissions may generate partisan rancor comparable to what we see from states where one party entirely controls the redistricting process and engages in a partisan gerrymander." Similarly, the Brennan Center report notes that "states that used a tiebreaker model popular in earlier reforms experienced much lower levels of satisfaction, mainly because the tiebreaker tended to end up siding with one party or the other, resulting in a winner-take-all effect."[12]

[11]Only Idaho (6 members) and Washington (4 members and 1 non-voting member) have perfectly balanced (by partisanship) independent commissions, and in these cases some bipartisan cooperation is needed for them to successfully create district maps, though researchers have illustrated that, in practice, balanced commissions may produce incumbent-protecting gerrymanders (McDonald 2004).

[12]McDonald (2004, p. 383) expresses similar qualms about tiebreakers: "Often, commissioners have strong common prior beliefs about the likely partisanship of the tiebreaker, and therefore balk at compromise during initial negotiations. Once chosen, the tiebreaker then sides with one of the parties and a partisan plan is adopted."

Last of all, creation of an independent redistricting commission is not a realistic option for citizens of many states. Many states do not have a legislative process allowing statutes or state constitutional amendments by initiative. Of the 24 states that do, eight (one-third) have independent redistricting commissions already. But the states with the most intense partisan gerrymandering seem unlikely to voluntarily relinquish authority over redistricting to an independent commission. For example in Maryland, North Carolina, Pennsylvania, Texas, and Virginia, voters cannot feasibly establish non-partisan independent redistricting commissions, since these states do not have an initiative process.

2.2 Other Proposed Solutions

Some of the most promising alternative solutions to gerrymandering draw inspiration from the cake-cutting problem; how do two people perform the fair division of a piece of cake without the need of third-party intervention? The solution is to arbitrarily choose one as the first mover; she divides the cake and then the second-mover may choose between either of the pieces. This logic, applied to geography, has inspired several redistricting proposals.

In Landau, Reid and Yershov (2009), the authors describe an interactive protocol whereby an independent third party divides the state into two and then each party negotiates over who gets to redistrict one section of the state. The parties each independently redistrict their agreed-upon parts of the state. Combining the two results in a final map.

In Pegden, Procaccia and Yu (2017), the authors propose a protocol where each of two parties alternate back and forth drawing district maps. Termed "I-cut-you-freeze," the protocol involves a back and forth where one party draws a map, the other party freezes in place one district from that map, and then redraws a new district map for the remaining area in the state. The players alternate between "cutting" and "freezing" until producing a full map.

Neither of these approaches has seen any take-up in the real world. The difficulties of implementing these solutions in practice are several-fold. The method in Landau, Reid and Yershov (2009) requires a neutral third party, which has proven to be a stumbling block in the past. Both the Landau, Reid and Yershov (2009) and Pegden, Procaccia and Yu (2017) approaches abstract from real-world geographies and do not place constraints on how voters are assigned to districts. Furthermore, because they involve multiple stages of bargaining between the parties, these approaches are impractical to simulate in real-world contexts using actual geographies and voter rolls. Thus, lack of information about implementation and potential results with real electoral geography and population information make it unlikely that decision makers would adopt these protocols.

Finally, Ely (2019) proposes a protocol with a similar "I-cut-you-freeze" flavor to Pegden, Procaccia and Yu (2017), but with an explicitly spatial addition to the process. The first party draws a full set of districts. Any district that is convex (i.e., a straight line can be drawn between any two points in a convex district) is locked into place. However, the second party has the ability to redraw any non-convex districts so that they are convex. This two-stage process assures the creation of a map without misshapen districts. However, this proposal also meets with some practical issues. First, in some states it is likely not possible to meet equal population requirements while also maintaining convex districts. Second, even states with convex districts can be extraordinarily biased in favor of one party, depending on the geographical distribution of voters (Alexeev and Mixon 2017).

In what follows, we describe the *Define-Combine Procedure*. The approach that we propose takes inspiration from the cake-cutting methods but retains several practical advantages.

3 The Define-Combine Procedure

Suppose a state (or city, school district, or other entity engaged in redistricting) with population P needs to be divided into N contiguous single-member districts with equal population P/N. Elections in the state are contested by two parties, A and B. We assume for simplicity that all people in the state vote in all elections, and their voting decision is based solely on their personal partisan preference; the makeup of their district and the candidates who run have no impact on their vote choice.[13] Let v_A be the number of votes in the state for Party A, and $v_B = P - v_A$ be the number of votes for Party B. For each district d, let v_{dA} and v_{dB} be the number of votes in the district for each party. A districting map M is thus a set of N districts, and each district is itself a set of

[13]The levels of turnout and voting are not essential to this game. The most important things are (1) that districting does not affect vote choice, and (2) each party can anticipate which districts they will win and lose (or the probability of winning and losing).

$^{P}/_{N}$ specific voters.[14] Thus, for this framework there is a *finite* set of possible maps $\mathcal{M}$ (though it grows extraordinarily large as the population P increases). And, for any map, both parties can determine the number of votes they will receive in each district and the number of districts that they will win.

We assume that both parties are seat-maximizers; their goal is to win as many of the N seats as possible in the next election. For any given map M, the utility of party A is

$$U_A(M) = \frac{\sum_{d=1}^{N} V_d}{N}$$

where

$$V_d = \begin{cases} 1 & \text{if } \nu_{dA} > \nu_{dB} \\ 1/2 & \text{if } \nu_{dA} = \nu_{dB} \\ 0 & \text{if } \nu_{dA} < \nu_{dB} \end{cases}$$

and the utility of party B is $U_B(M) = 1 - U_A(M)$. Both parties are risk-neutral; they are indifferent between winning one district and tying in two districts (with a 50% chance of winning the election in each). U_i is equivalent to the percentage of seats won by party i.[15]

3.1 The Unilateral Process

We consider two methods of drawing district maps. In the first method, one party has unilateral control of the process. Given a set of potential valid maps $\mathcal{M}$, when party i draws the maps, it will select a map $\hat{M} \in \mathcal{M}$ that maximizes U_i.[16] Under this method of redistricting, we should expect that, where possible, party i will maximize their partisan advantage relative to party j by strategically cracking and packing party j's voters to minimize the seats won by party j. In many

[14] For example, if we were to enumerate the full set of voters $1, 2, \ldots, P$ then a hypothetical district 1 could be represented as district $1 = \{1, 4, 8, 23, \ldots, 974, P\}$

[15] We also assume that there is no uncertainty in future elections and that voters don't change their partisanship over time or move across districts (i.e. that maximizing seats won in the next election is equivalent to maximizing seats won in all elections with that map).

[16] In practice, there will be a large number of maps in $\mathcal{M}$ (even in some of the simple examples here, there may be millions or billions of legal maps). Thus, $\mathcal{M}$ does not have to be the complete set of feasible maps, but rather a subset of all feasible maps in which U_i varies.

cases, it will be possible for party i to win a substantially larger share of the seats than its statewide vote share.

This method, which we will call the "unilateral process," approximates the redistricting process in states where one party controls redistricting for a given map.[17] The party in control seeks to maximize the number of seats they will win in the next election. While other factors, such as incumbency or vote margins in close seats, may factor into their districting decisions, in most places where we observe efforts to gerrymander for partisan gain, the party in power appears to choose a map that maximizes seats won. For example, when independent experts have used map-drawing software to simulate thousands of possible maps in a state, the observed maps in states with a unilateral process appear to be among the most partisan possible.[18]

3.2 The Define-Combine Procedure

The second method we consider is our own innovation, the two-stage Define-Combine Procedure. In this model, the power to draw the map is divided between the two parties (i.e., the players in the game are Party A and Party B), but in each stage of the process one party acts unilaterally.

Suppose party A acts in the first stage as the "Definer," and party B acts in the second stage as the "Combiner." The game proceeds as follows:

1. Party A defines a set of $2N$ contiguous, equally populated districts. To avoid confusion with the following stage, we refer to these districts as *subdistricts*.[19]

2. Party B creates the final map of N districts by combining together pairs of 2 contiguous subdistricts.

Party A moves first and so has a strategy profile consisting of a selection of a map $M_A \in \mathcal{M}$, the set of all maps with $2N$ valid districts.[20] Party B combines subdistricts to create a map $M_B \in Q(M_A)$, where $Q(M_A)$ is the set of valid groupings of the subdistricts in M_A.

[17]Different parties may control the redistricting process for different maps. For example, if each legislative chamber controls its own process, and each chamber has a different majority party, then each party will control the process for one map.

[18]See Chen and Rodden (2015) for Florida, Cho and Liu (2016) for Maryland, Chen (2017) for Wisconsin, Magleby and Mosesson (2018) for Virginia, and Duchin (2018*b*) for Pennsylvania; all argue that chosen plans in states with unilateral processes are extreme outliers as compared to the set of simulated possible maps.

[19]A related game could involve defining kN subdistricts where k is a positive integer greater than 2.

[20]Valid districts are contiguous (except where required by geographic features such as islands) and have equal

Party B, the second mover, will select a best-response to any proposed set of subdistricts. The strategy σ_B is a mapping from the set of valid groupings of sub-districts to a single map, M_B, such that

$$\hat{M}_B \equiv \sigma_B(M_A) \in \text{argmax } U_B(Q(M_A)).$$

Because voters themselves are indivisible and the districts in this setup consist of sets of voters, any game has a fixed number of possible districts. Also, the second-mover knows what the first mover has chosen to do. In a finite extensive game with perfect information, such as Define-Combine, there exists a subgame perfect equilibrium (Osborne 2004, p. 173). Furthermore, it can be solved using backward induction. For the map M_A selected by party A, party B will examine all of the possible maps it could draw, $Q(M_A)$. From these possibilities, it will select the map, $\hat{M}_B \in Q(M_A)$ which maximizes the percentage of the seats won by party B.[21] For every possible map $M_A \in \mathcal{M}$, Party A can anticipate what ultimate map M_B party B would draw. Therefore, it selects the map $\hat{M}_A$ that maximizes the percentage of seats won by party A subject to party B's best-response pairings, with payoff $U_A(\sigma_B(\hat{M}_A))$.[22]

3.3 A Simple Example

A simple example illustrates the Define-Combine framework. Consider a map, as in Figure 1, with a population divided equally in support of Party A (white circles) and Party B (black circles). Each precinct (a circle) is occupied by a single voter. Suppose redistricting requires that the parties divide this area into five equally populated contiguous districts, each with six voters. This simple case, with only thirty voters and five districts, yields 33,344 valid possible maps.

When Party A acts unilaterally, the party maximizes its own number of seats and gerrymanders such that Party A wins three districts, ties in one district and loses one district. Figure 2(a) shows one such map (out of many equivalent possibilities) drawn by Party A under unilateral power.

population. We do not impose any compactness, split geography, or other restrictions, but such limitations could be included here. The one exception to this is that valid districts may not include "donuts," where one district entirely encircles another. A map consisting only pairs of encircled subdistricts entirely remove the ability of the combining party to have any choices when combining subdistricts, and therefore violate the rules of this procedure.

[21] In practice, there may be many maps that equally maximize the percentage of seats.

[22] Equivalent to minimizing the percentage of seats won by opposing Party B.

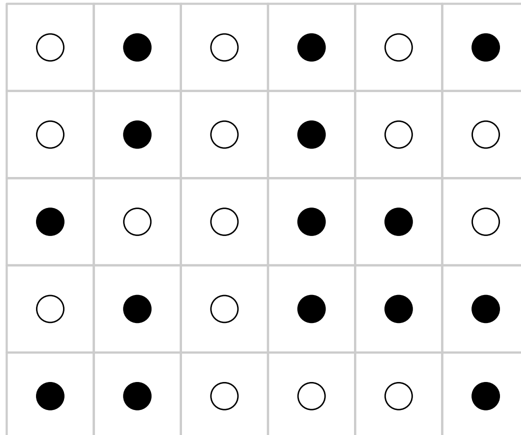


Figure 1: Example Distribution of Voters for a Simple Grid.

In this example, Party A packs Party B voters into district 4 (light blue) and wins majorities in districts 2, 3, and 5, and ties in district 1 (dark blue), with $U_A = \frac{7}{10}$ and $U_B = \frac{3}{10}$.

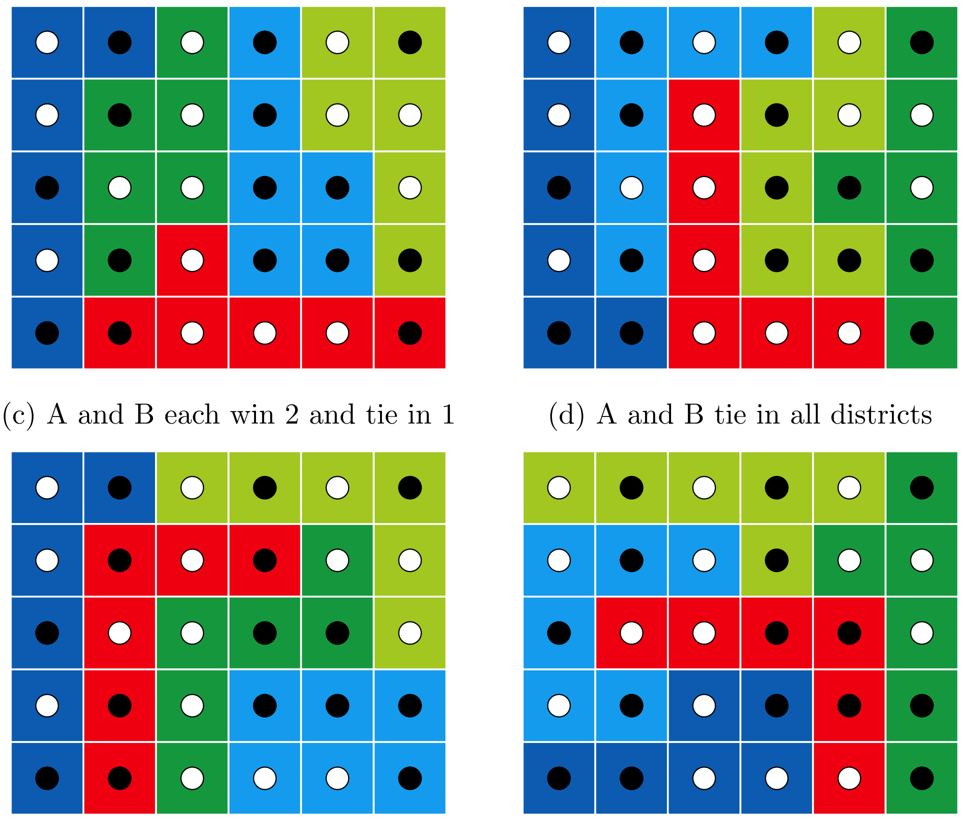
When Party B acts unilaterally, the party follows a similar strategy and also wins three districts, ties in one, and loses in one (see Figure 2(b)). The top right panel shows a map that could be selected by party B if the party drew the map unilaterally. Party B packs party A's voters in district 2, conceding that district but winning in districts 3, 4, and 5, and tying in district 1 ($U_A = \frac{3}{10}$ and $U_B = \frac{7}{10}$).

Alternative protocols can achieve fairer maps in this example. For example, Figures 2(c) and 2(d) both provide examples of district maps that result in proportional representation for both parties ($U_A = U_B = 0.5$).

Looking at the full set of possible maps, Party A wins a majority of the seats (either 3-2 or 3-1-1) in 12% of all possible maps, and Party B wins a majority in 6% of the maps.[23] In the remaining 82% of the maps, the two parties win the same number of seats. While there is a large number of maps that do not advantage either party, we would not expect to see these maps selected in a redistricting process where one party unilaterally chooses the map.

We now apply the Define-Combine protocol to this simple example. In the first stage, the defin-

[23] The lack of symmetry here is a result of this particular distribution of voters. Under other distributions, Party B may have more maps that provide an advantage. Regardless, only one such map is necessary for each party to select a map favoring itself over the other party.



(a) Best map for Party A
A wins 3, ties 1, loses 1

(b) Best map for Party B
B wins 3, ties 1, loses 1

(c) A and B each win 2 and tie in 1

(d) A and B tie in all districts

Figure 2: Examples of Valid Maps for a Simple Grid. White circles represent voters supporting Party A, and black circles represent voters supporting Party B.

(a) Division with 2 possible combinations

(b) Division with 8 possible combinations

(c) Division with 25 possible combinations

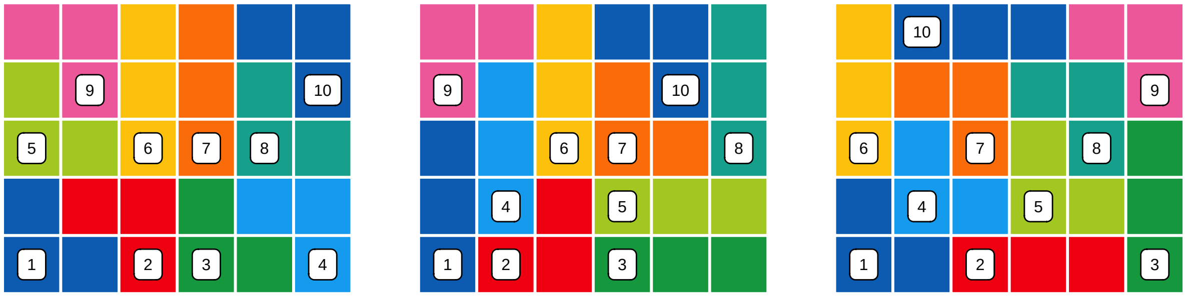


Figure 3: Examples of Defined Subdistricts for a Simple Grid

ing party will draw a map consisting of ten contiguous subdistricts, each with three voters. There are 8,342 valid divisions of this grid. In the second stage, the combining party selects contiguous pairs of subdistricts to create the final district map. The number of possible combinations in the second stage varies based on the subdistricts defined in the first stage. In this example, the number of combinations varies from 2 to 25 possibilities, with an average of 8.5 combinations. Figure 3 presents three examples of defined subdistricts, with varying numbers of possible combinations.

Consider the defined subdistricts presented in Figure 3b. Using this voter distribution and these subdistricts, the the combiner can choose among eight possible pairings, which produce very different electoral outcomes: Party A wins a majority of seats in 2 cases, ties in 3 cases, and holds a minority of seats in 3 cases. Figure 4 presents four of these possible pairings. If Party B moved first and selected the map of subdistricts displayed in Figure 3b, then Party A would respond by choosing the top left map (a) in the combine stage – winning three districts, losing one and tying one. If Party A had moved first, then Party B would respond by selecting the bottom right map (d) in the combine stage – winning three districts and losing two.[24]

For any possible proposed map (i.e., for each sub game), the defining party analyzes the resulting combinations and determines the best-response for the combining party. The defining party chooses the map that minimizes the maximum utility that the combining party gets from making the optimal pairing in the sub game. Given the distribution of voters in our running example, Figure 5 presents

[24]Equivalently, when Party B is the Combiner they could also choose a pairing (not shown in the Figure) yielding a win in two districts, a loss in one district, and a tie in two districts.

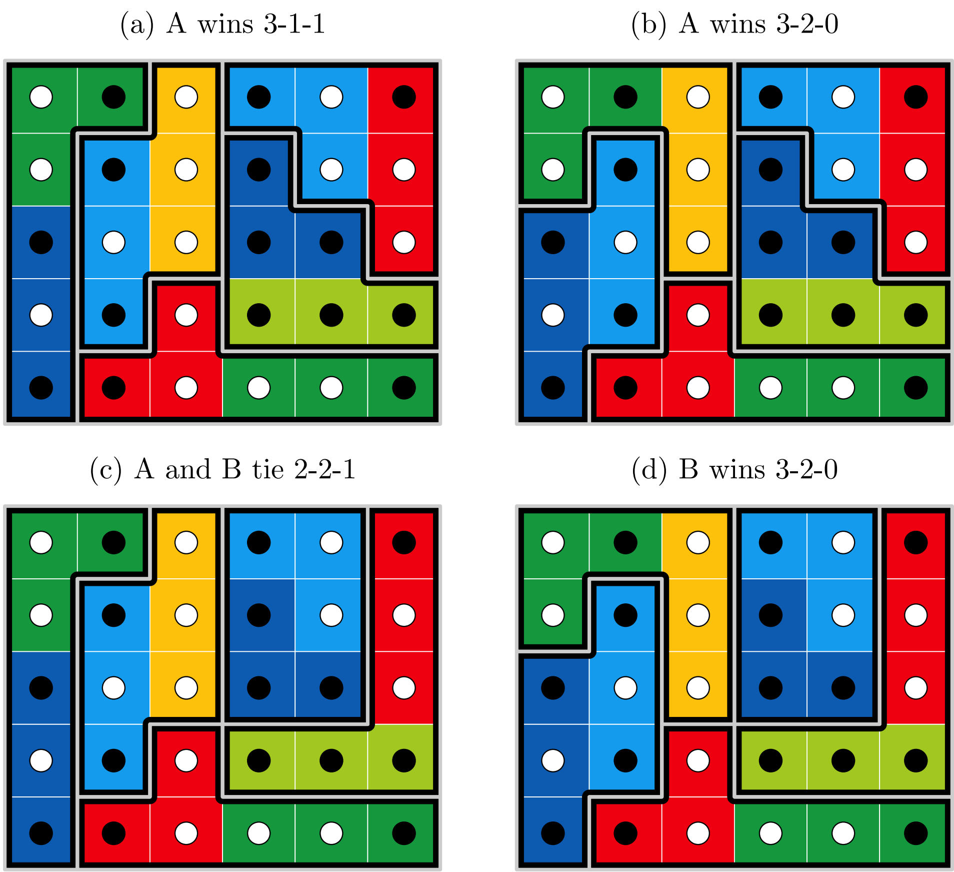


Figure 4: Combinations of an Example Defined Map

(a) A Defines; B Combines (b) B Defines; A Combines

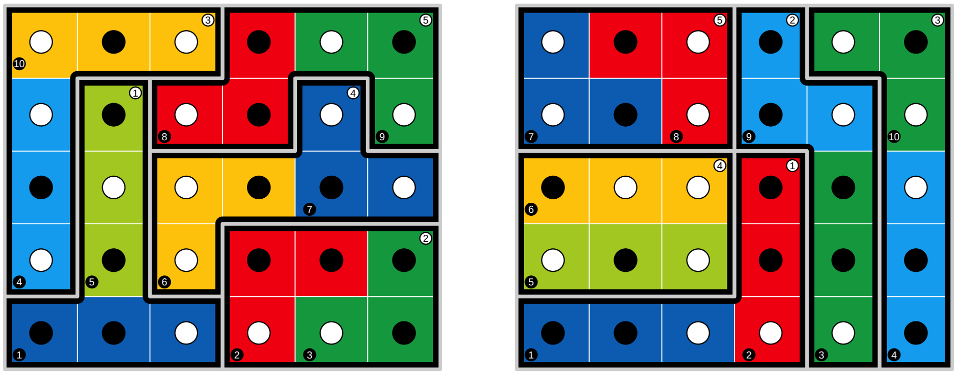


Figure 5: Define-Combine Procedure Results for a Simple Grid

the results of the Define-Combine procedure if Party A goes first, on the left, and if Party B goes first, on the right. In this example, there are multiple equilibria and we present just one graphically. The defined subdistrict plan selected by Party A results in each party winning two seats and tying in one. Party B cannot choose any other combination of these subdistricts to improve the outcome. If Party B moves first, then in equilibrium both parties again win two seats and tie on one. Thus, both maps resulting from this procedure result in a less biased districting plan as compared to the unilateral process. While in this example both parties get the same payoffs regardless of who moves first, such an outcome is not guaranteed. In the next section, we extend this example by showing that this pattern persists across a wide sample of voter distributions, and we illustrate how the results change as the partisan balance in the state changes.

3.4 Evaluating the Define-Combine Procedure with Hypothetical Maps

The example above illustrates how the Define-Combine Procedure produces less biased maps for one particular distribution of voters. We now extend the analysis by simulating thousands of different distributions of voters and identifying the maps selected by each party when choosing maps unilaterally and when using Define-Combine. The simulations proceed in four steps:

1. Define a grid of P precincts; each will have the same population.

2. Generate a random distribution of voters in each precinct. Instead of making each precinct either one Party A voter or one Party B voter, each precinct contains the same population

size, but with a randomly selected percentage of voters supporting each party. First, we pick a target vote share m for Party A in the grid as a whole. We vary this across simulations in 10% increments from 20% to 80%. For each target vote share, we draw a vote share for each precinct from a truncated normal distribution with mean m.[25] We repeat this process 500 times for each level of m, resulting in 3,500 different distributions of voters.

3. Generate potential maps for the grid:

   (a) Generate a set of possible maps of N districts, and a set of possible maps of $2N$ districts. For the simple grid above, we generated every possible map. For more complex grids, we generated a random sample of maps.

   (b) For the set of $2N$ districts, generate all possible plans that combine pairs of contiguous districts. For the simple grid, we generated every possible combination, and for more complex grids we generated a random sample of combinations.

4. For each distribution of voters, examine the set of generated maps to identify:

   (a) The best map for Party A, if Party A chooses a map unilaterally.

   (b) The best map for Party B, if Party B chooses a map unilaterally.

   (c) The map Party A would choose if it goes first under the Define-Combine Procedure.

   (d) The map Party B would choose if it goes first under the Define-Combine Procedure.

   For each identified map we calculate the number of seats won by each party.

Figure 6 presents the results for the 30-voter grid used in the previous examples. The x-axis corresponds to the percentage of voters supporting Party A, and the y-axis to the percentage of seats won by Party A. The dotted lines show the average across simulations for when Party A (in blue) and Party B (in red) draw the district map unilaterally. At the tails, when one party dominates overall vote share, the results remain similar no matter who controls the unilateral process. However, when both parties are competitive in terms of vote share, the share of seats won

[25] The truncated normal distribution is bounded at 0 and 1 and has a standard deviation of 0.25.

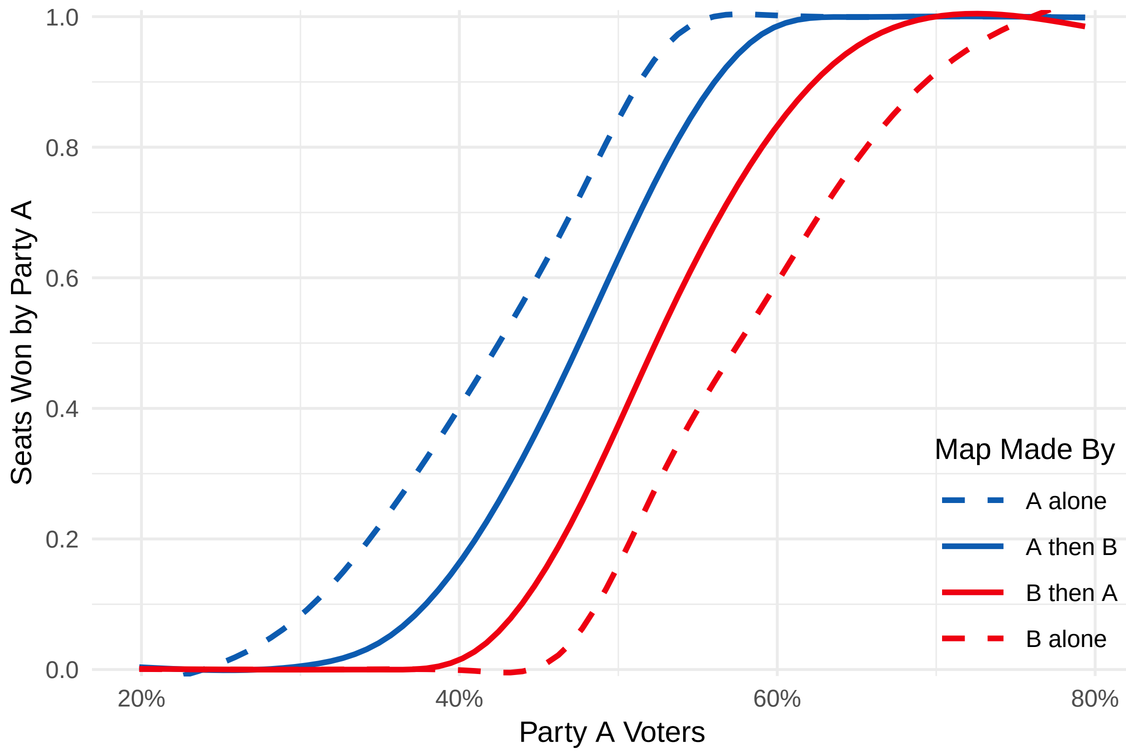


Figure 6: Results for Voter Distribution Simulations on a Simple Grid. Each simulation uses a 5x6 grid.

by Party A varies substantially based on who draws the maps. When the parties have the same number of voters in the state, whichever party draws the map unilaterally can win on average four of the five seats. The solid lines show the average results of the Define-Combine Procedure, with the blue line corresponding to Party A as the definer followed by Party B as the combiner, and the red line the reverse. Under Define-Combine, the defining party's advantage declines by a full seat on average as compared to drawing the map unilaterally.

Figure 6 also demonstrates that the Define-Combine procedure offers the defining party a first-mover advantage in this context; on average the definer wins about one additional seat over the combiner. However, this advantage declines as the size of the grid and/or the number of districts increases. Figure 7 displays the average number of seats won by Party A under each of the four methods described above, but for a larger hexagonal grid where 150 precincts are divided into 15 districts. While there is a substantial gap between the maps drawn unilaterally by each party, the

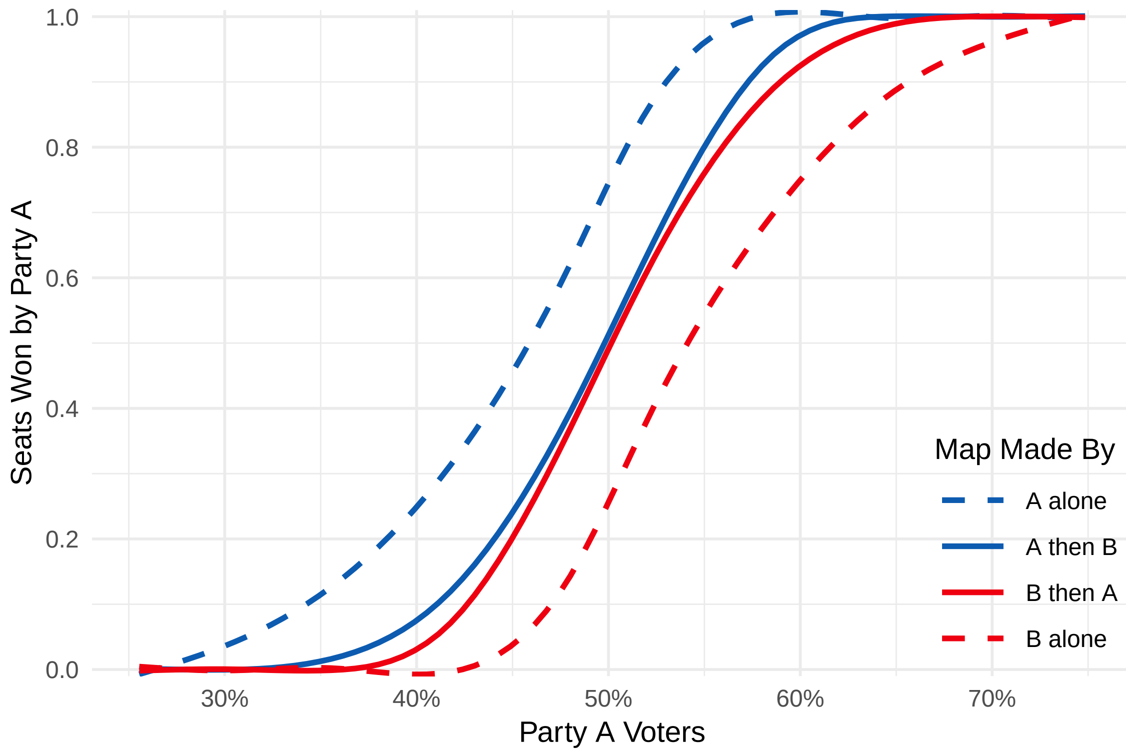


Figure 7: Results for Voter Distribution Simulations on a Larger Grid (150 precincts; 15 districts)

seats won nearly converges under the Define-Combine Procedure when the state is competitive, regardless of which party goes first. The first mover retains a very small advantage, of about 1% of the seats, or .15 seats in this example.

4 Evaluating the Define-Combine Procedure Using Simulated Maps

Having demonstrated the Define-Combine Procedure and its outcomes on hypothetical grid maps, we now turn to applying it to real maps and election data in the United States. We selected eight states, varying in size and partisanship, and used map-drawing algorithms to generate thousands of possible maps of $2N$ subdistricts and random samples of combinations of these districts. Using this sample, we identified the maps that would be chosen by each party unilaterally and by each party in sequence under the Define-Combine Procedure, based on election results from the 2016 presidential election. This process allows us to assess if the Define-Combine Procedure would actually reduce the partisanship of legislative maps in practice.[26]

Simulating valid (contiguous, roughly equally populated) districting maps represents a substantial computational challenge, and scholars have recently developed several different algorithms to do so.[27] We selected the `redist` package, developed by Fifield et al. (2019), which uses Markov Chain Monte Carlo simulations with parallel tempering to generate maps that theoretically span the full distribution of possible valid maps. We selected states based on availability of data, using shapefiles and election data from the Metric Geometry Gerrymandering Group (MGGG).[28]

For each state, we used the Fifield et al. (2019) algorithm to generate 10 independent Markov chains of 10,000 maps of $2N$ subdistricts, where N is the number of congressional districts in that state, resulting in 100,000 maps per state.[29] We thinned the chain by selecting 5% of the simulated maps and then generated 100 possible combinations of pairs of districts to evaluate.[30] Using actual election results, we then identified the best map for the Democrats and for the Republicans, if they were to draw the map unilaterally, and the maps that would result from the Define-Combine Procedure if each party were to go first. Figure 8 displays the results, using the actual votes cast in the 2016 presidential election.

In all eight states, there is a multiple-seat gap between the number of seats won by Democrats when Democrats draw the map unilaterally and when Republicans draw the map unilaterally. Under the Define-Combine procedure, the gap is reduced to one or zero seats in every state, except in Virginia, where Democrats have a two-seat advantage. In every state, the Define-Combine Procedure results in a map where the outcome, regardless of which party defined and which party combined, is in the middle of the range between the unilateral outcomes. The Define-Combine

[26]The results in this section are preliminary. We are continuing to improve the map-drawing simulations to ensure that we are able to analyze a large and representative sample of potential maps for each state.

[27]For examples of different algorithms and redistricting simulation software, see (Altman and McDonald (2011); Chen and Rodden (2013), later modified in Chen and Cottrell (2016); Cho and Liu (2016); MGGG (2018); Magleby and Mosesson (2018); and Fifield et al. (2019).

[28]Precinct shapefiles and election data available from: `https://github.com/mggg-states`.

[29]Maps are drawn and evaluated using precinct-level population data and election results. In each chain we restricted population deviations to 20% (a level commonly used in the literature (Fifield et al. 2019)), used a different set of initial districts (based on dividing the current map's districts into two districts), and used parallel tempering to increase the probability of simulating districts across the full possible distribution. Full replication code available upon publication.

[30]For some of the maps, particularly in states with fewer congressional districts, there were not 100 possible combinations of districts for every defined map. In these cases we used all possible unique combinations.

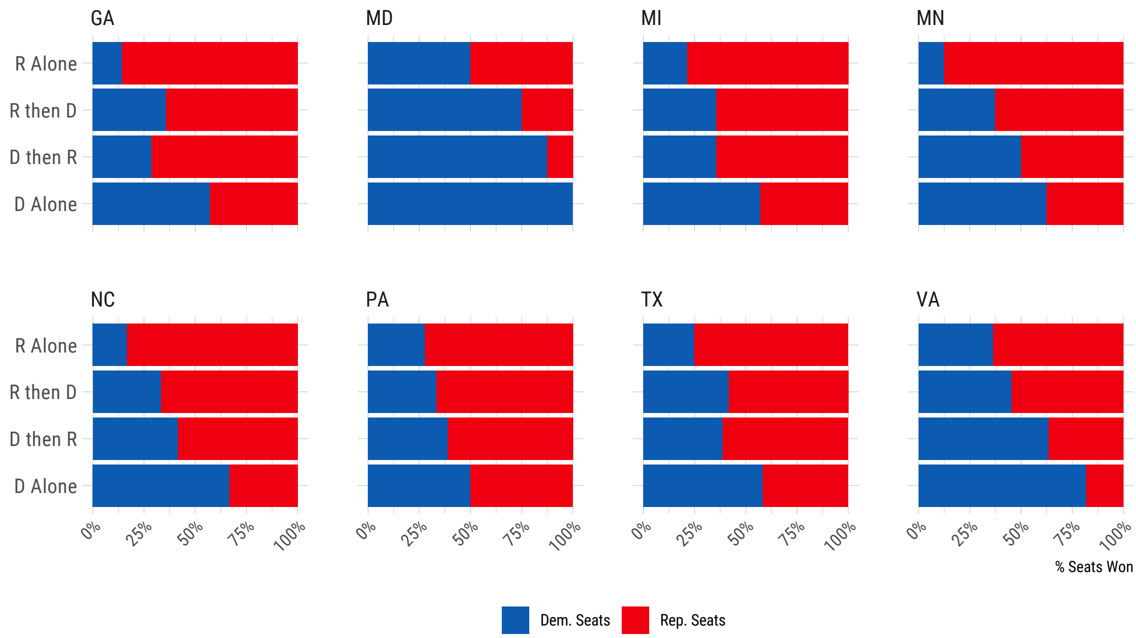


Figure 8: Results Using Simulated Maps and 2016 Presidential Election Results

Procedure restricted the ability of each party to gerrymander. However, the resulting map may still favor a particular party. For example, in Michigan, the best Democratic map produced 8 Democratic wins out of 14 districts, while the best Republican map produced 11 Republican wins. Under the Define-Combine procedure, the resulting maps (regardless of first-mover), produced five Democratic seats and nine Republican seats, even though both parties received about 50% of the vote in the 2016 presidential election. This may be due to the geographic distribution of people and voters across the state, where Democrats are concentrated in urban areas, creating an unintentional advantage for Republicans.[31]

The simulations produce similar results if parties hold objectives other than maximizing simple win totals. For example, parties might try to distribute supporters as efficiently as possible by maximizing the efficiency gap in favor of their party (Stephanopoulos and McGhee 2015). Al-

[31]Past research such as Chen and Rodden (2013) has pointed out how the spatial clustering of one party can lead to an increased likelihood of facing an electoral disadvantage, even when drawing maps randomly (i.e., without an explicit partisan bias). In addition, more simulations with more rigid restrictions on population equality and compactness might further reduce the Republican advantage that we have observed.

ternatively, parties might seek the safest majorities possible by maximizing the vote share in the median district, relative to the state as a whole (i.e., maximizing mean-median difference (McDonald and Best 2015) when selecting maps). When we use these objectives in the simulations, the moderating effects of Define-Combine persist. When one party acts alone, the magnitude of the efficiency gap or mean-median distance remains much larger than when the parties redistrict under the Define-Combine Procedure. Considering that both of these measures are commonly used to identify partisan gerrymanders, the Define-Combine procedure effectively produces less gerrymandered district maps.

5 Implementing the Define-Combine Procedure

Having introduced the Define-Combine Procedure and demonstrated that it produces less partisan maps in both hypothetical simulations and using real state geography and election results, we now turn to how it could be implemented for redistricting in practice.[32]

Both the hypothetical grid simulations and the simulated maps using real data produce the same core result: the Define-Combine Procedure produces less partisan maps than those drawn unilaterally by either party. In the hypothetical grids, the defining party received a small advantage on average. When voters split their support evenly between each party, the defining party wins slightly more seats. For example, with a 30-person grid divided into 5 districts, the defining party won three seats on average under Define-Combine, compared to four seats when drawing the map unilaterally. As the size of the grid and the number of districts drawn increased, this advantage substantially diminished. In the simulated maps using real data, the first-mover advantage is less clear. In many states, each party preferred to be the defining party, but in two cases, Texas and Georgia, the combining party won slightly more seats. This may be a function of the state's particular electoral geography, the elections analyzed, or the set of simulated maps generated by the map-drawing algorithm.

Assigning the roles of the defining and combining parties is a key part of the implementation process. States could assign the order randomly, alternate between parties each districting cycle, or

[32]While our discussion in this section focuses on the fifty states, this should be generally applicable to local redistricting as well, but with the caveat that changes to state law may be required when the local redistricting process is set by state law.

employ partisan factors, such as the majority party in the legislature or the party of the governor. Assigning roles based on control of the legislature or state offices may offer an advantage to adopting this procedure. If the majority party currently enjoys a procedural advantage in the redistricting process, they may be more willing to accept Define-Combine as a reform if they can choose the role that will still offer them an edge, even though adopting Define-Combine will diminish their advantage.

In states where the legislature controls redistricting, the legislature could adopt the Define-Combine Procedure in several different ways. Most straightforwardly, if only legislative rules govern the current process, the legislature could change the rules and require that the committee responsible for drawing maps use Define-Combine. Further rule changes might require that a map produced by this process not be amended on the floor or by the other chamber. If a partisan commission controls a state's redistricting, a legislature might have to pass a law requiring that the committee use this process. Similarly, in states with bipartisan or non-partisan commissions, a state law could require use of Define-Combine.

Finally, and perhaps most plausibly, courts could use this process when ordering remedial maps — rather than ordering the state legislature to draw a new map under specific guidelines, or selecting a special master to draw a new map. For example, in *Common Cause v. Lewis*, a North Carolina court ruled that the state legislative maps were unconstitutional, noting that the map does "not permit voters to freely choose their representative, but rather representatives are choosing voters based upon sophisticated partisan sorting."[33] The court ordered that the state legislature draw remedial maps in a non-partisan manner, but did not specify the exact process to be used. The Define-Combine Procedure would be a simple and efficient framework for future cases like this one.

6 Conclusion

The Define-Combine Procedure features simple rules, clear strategies for each party, and an efficient implementation framework for small and large numbers of districts alike. One obstacle to implementation for past theoretical approaches has been the difficulty for decision makers in either party to predict outcomes in the real world. Define-Combine, as we have demonstrated, can be applied

[33] *Common Cause v Lewis*, No. 18 CVS 014001, 2019 WL 4569584, at *3 (N.C.Super. Sep. 03, 2019).

to real-world geographic data, which allows analysts to predict outcomes and reduce uncertainty surrounding the redistricting process.

Political parties in power will always oppose ceding it, but this is doubly so when the choices they face require embracing significant uncertainty about future political outcomes. Because Define-Combine is a two-stage game and existing computing resources can help solve it, our proposal represents a step towards providing an alternative mechanism to legislature-based or commission-based redistricting that is actually feasible to implement. At the same time, the framework gives parties the autonomy to respect communities of interest, geographic boundaries, and other political concerns – that is, to internalize the wide range of factors that play important roles in decisions about redistricting – while nonetheless tempering the partisan bias that tends to emerge during redistricting. By involving both parties but setting them in opposition to each other, rather than requiring bipartisan cooperation or independent third-party mediators, Define-Combine offers a partisan solution to the extraordinarily partisan process that is redistricting.

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