A Partisan Solution to Partisan Gerrymandering: The Define-Combine Procedure

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Redistricting reformers have proposed many solutions to the problem of partisan gerrymandering, including non-partisan commissions and bipartisan commissions with members from each party. Redistricting litigation frequently ends with court- or special-master-drawn plans. All of these methods require either bipartisan consensus or the agreement of both parties on the legitimacy of a neutral third party to resolve disputes. In this paper we propose a new method for drawing district maps, the Define-Combine Procedure, that substantially reduces partisan gerrymandering without requiring a neutral third party or bipartisan agreement. One party defines a map of 2N equal-population contiguous districts. Then the second party combines pairs of contiguous districts to create the final map of N districts. Using real-world geographic and electoral data, we use simulations and map-drawing algorithms to show that this procedure dramatically reduces the advantage conferred to the party controlling the redistricting process and leads to less biased maps without requiring cooperation or non-partisan actors.

Later this year, every state in the U.S. will begin redrawing their Congressional and legislative district boundaries. Many city councils, school boards, county commissions, and other representative bodies will also draw new maps. While redistricting is required by law in order to equalize population across districts, it also creates opportunities for political actors to benefit certain groups over others. In particular, partisan gerrymandering is used to advantage one political party over the other, even enabling a party to win a majority of the seats in a legislative chamber without winning a majority of votes in the election. For example, in 2016, Republican candidates for the Wisconsin State Assembly won only 45% of the state-wide vote but, due to partisan gerrymandering, won 63 of 99 seats (64%) and maintained control of the chamber.

Gerrymandering hinders democratic representation. The public in a representative democracy expects “continued responsiveness of the government to the preferences of its citizens” (Dahl, 1971), a relationship fostered by electoral competition—specifically, the threat of removing from office lawmakers not in step with their constituents (Canes-Wrone, Brady, and Cogan, 2002). Gerrymandering becomes a critical problem when parties undermine electoral competition by drawing districts that create, enhance or lock in a partisan advantage.

Reformers have cycled through a plethora of solutions for drawing new district boundaries, including bi-partisan commissions with members from each party, non-partisan geographers, court- or special-master-drawn maps, and court-ordered non-partisan remedial maps drawn by a state legislature. All these schemes either require a neutral third-party actor, trusted by both political parties, to select a fair map, or they necessitate that opposing parties cooperate. However, in today’s hyper-partisan environment, there are few independent third-party actors considered by both sides as able to fulfill this role legitimately and attempts at cooperation have generated acrimony along with maps viewed as partisan or incumbent-protecting gerrymanders. In short, intense controversies continue to surround redistricting, even in states that have enacted reforms.

Here we propose a new method for drawing maps, the Define-Combine Procedure, which requires neither a neutral third party nor bipartisan cooperation. We develop a simple framework that allows each party to act in their own partisan self-interest but nonetheless achieves a significantly fairer map than would be drawn by either party on its own. We divide the redistricting process into two stages. Suppose a state must be divided into N equally populated districts. First, one party—the “definer”—draws 2N contiguous, equal population districts. Then, the second party—the “combiner”—selects contiguous pairs of districts from the set defined by the first party to create the final districts. This method produces N equally populated, contiguous districts. By dividing the responsibility of drawing districts into two separate stages, in which each party retains complete autonomy in their own stage, the parties counteract each other’s partisan ambitions while still maintaining considerable flexibility to achieve other objectives, including maintaining compactness, contiguity, and communities of interest. We show that the Define-Combine Procedure (DCP) produces maps with large reductions in bias advantaging either political party as compared to maps drawn unilaterally. Using simulations based on real-world geographic and electoral data, we assess DCP’s performance in eleven states—to our knowledge, the first effort demonstrating such improvements across a wide variety of geographic and electoral contexts.

Limitations of Current Partisan Gerrymandering Fixes

Reformers have sought to rein in partisan gerrymandering by shifting redistricting authority from partisan legislatures to nonpartisan independent redistricting commissions. State legislatures draw districts in over half of the
U.S. states, with the remainder relying on commissions (see Table S1). While redistricting registers higher levels of citizen satisfaction in states with commissions, commissions do not represent a catch-all solution. The absolute level of citizen satisfaction with redistricting remains low even in commission states (Schaffner and Ansolabehere, 2015). Furthermore, a majority of commission-based states are not actually independent from interference by a state legislature, and those that are rely on a member or members to act neutrally, sometimes touching off controversy—as when the Governor of Arizona sought to impeach the independent chair of her state’s commission (Druke, 2017). Last of all, states without an initiative process appear unable or unlikely to implement commission-based redistricting, which would require a state legislature to cede its authority.

Another common fix to partisan gerrymandering is through court intervention. While federal judicial intervention over partisan gerrymandering claims was effectively closed by the Supreme Court’s decision in Rucho v. Common Cause (2019), anti-gerrymandering efforts in state courts have had some recent success.1 These cases rely on state constitutions that either ban partisan gerrymandering outright or include some version of a “free election” clause. Intervention by state courts on a state-by-state basis will not provide a comprehensive solution, however, since many state constitutions place little or no restrictions on state legislatures’ power to redistrict. More generally, litigation over redistricting remains commonplace and courts have struggled to adjudicate partisan gerrymandering disputes due to unresolved questions over standards for measuring and comparing partisan gerrymanders. SI Appendix A.1 provides a comprehensive discussion of the challenges presented by redistricting in legislatures and commissions as well as redistricting litigation.

Given these challenges, researchers have previously proposed solutions to partisan gerrymandering, with some of the most promising drawing inspiration from the cake-cutting problem. While this logic, applied to geography, has inspired several redistricting proposals Landau, Reid, and Yershov (2009); Pegden, Procaccia, and Yu (2017); Ely (2019); Alexeev and Mixon (2017); Brams (2020), none of the existing work has addressed implementation across different states or electoral contexts using real-world data. The complexity of most previous proposals also precludes real-world application, or even simulation using modern computing techniques. See SI Appendix A.2 for a comprehensive accounting of existing proposals.

DCP contrasts with the existing proposals in several ways. First, it does not require either forging bipartisan support between parties or appointing a third-party arbiter to resolve points of difference. A procedure that sidesteps these common stumbling blocks could reduce the contentious political disputes that accompany decennial redistricting, leading to fairer and more quickly-produced maps. Second, DCP is a process-based solution to partisan gerrymandering; it does not require courts to adjudicate controversial claims of partisan gerrymandering. As a result, state courts could use this approach to resolve redistricting disputes without having to define a standard for identifying partisan gerrymandering in the first place, thereby reducing lengthy and expensive litigation. Third, DCP is simple and could be implemented efficiently. Several other proposed solutions have appealing game-theoretic properties but in practice require multiple rounds of bargaining or map drawing and are difficult or impossible to solve computationally; in contrast, DCP’s two-stage process efficiently produces a complete and valid districting plan.

### The Define-Combine Procedure

Suppose a state (or city, school district, or other entity engaged in redistricting) with population \( P \) needs to be divided into \( N \) contiguous single-member districts with equal population \( P/N \). Elections in the state are contested by two parties, A and B. We assume for simplicity that all people in the state vote in all elections, and their voting decision is based solely on their personal partisan preference; the makeup of their district and the candidates who run have no impact on their vote choice.2 Let \( v_A \) be the number of votes in the state for Party A, and \( v_B = P - v_A \) be the number of votes for Party B. For each district \( d \), let \( v_{dA} \) and \( v_{dB} \) be the number of votes in the district for each party. A districting map \( M \) is a set of \( N \) districts, and each district is itself a set of \( P/N \) specific voters. Thus, for this framework there is a finite set of possible maps \( M \) (though it grows extraordinarily large as the population \( P \) increases). And, for any map, both parties can determine the number of votes they will receive in each district and the number of districts that they will win.

We first assume that both parties are seat-maximizers; their goal is to win as many of the \( N \) seats as possible in the next election3:

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2The levels of turnout and vote choice are not essential to this game. The most important things are (1) that districting does not affect vote choice, and (2) each party can anticipate which districts they will win and lose (or the probability of winning and losing). Additionally, our baseline example abstracts away from the incumbency advantage, but it could be incorporated as well into voter decision-making.

3Later in the paper we consider instances where parties may have other goals. For example, parties might seek to maximize bias, or they might have preferences over the level of responsiveness in the plan Katz, King, and Rosenblatt (2020).
where
\[ s_d = \begin{cases} 
1 & \text{if } \nu_{dA} > \nu_{dB} \\
\frac{1}{2} & \text{if } \nu_{dA} = \nu_{dB} \\
0 & \text{if } \nu_{dA} < \nu_{dB}
\end{cases} \]
and the utility of Party B is \( U_B(M) = 1 - U_A(M) \). Both parties are risk-neutral; they are indifferent between winning one district and tying in two districts (with a 50% chance of winning the election in each). \( U_i \) is equivalent to the percentage of seats won by party \( i \).

The Unilateral Redistricting Process. We consider two methods of drawing district maps. In the first method, one party has unilateral control of the process. Given a set of potential valid maps \( \mathcal{M} \), when party \( i \) draws the maps, it will select a map \( \tilde{M}_i \in \mathcal{M} \) that maximizes \( U_i \).

Under this method of redistricting, we should expect that, where possible, party \( i \) will maximize partisan advantage relative to party \( j \) by strategically cracking and packing party \( j \)'s voters to minimize the seats won by party \( j \). In many cases, it will be possible for party \( i \) to win a substantially larger share of the seats than its statewide vote share.

This method, which we will call the “unilateral redistricting process” (URP), approximates the redistricting process in states where one party controls redistricting for a given map. The party in control seeks to maximize the number of seats it will win in the next election. While other factors, such as incumbency or vote margins in close seats, may factor into districting decisions, in most places where we observe efforts to gerrymander for partisan gain the party in power appears to choose a map that maximizes seats won. For example, when independent experts have used map-drawing software to simulate thousands of possible maps in a state, the observed maps in states where redistricting is controlled by a single party appear to be among the most partisan possible.

The Define-Combine Procedure. The second method we consider is our own innovation, the two-stage Define-Combine Procedure. In this model, the power to draw the map is divided between the two parties (i.e., the players in the game are Party A and Party B), but in each stage of the process one party acts unilaterally.

Suppose Party A acts in the first stage as the “Definer,” and Party B acts in the second stage as the “Combiner.” The game proceeds as follows:

1. Party A defines a set of \( 2N \) contiguous, equally populated districts. To avoid confusion with the following stage, we refer to these districts as subdistricts.

2. Party B creates the final map of \( N \) districts by combining together pairs of 2 contiguous subdistricts.

Party A moves first and so has a strategy profile consisting of a selection of a map \( M_A \in \mathcal{M} \), the set of all maps with \( 2N \) valid districts. Party B combines subdistricts to create a map \( M_B \in Q(M_A) \), where \( Q(M_A) \) is the set of valid groupings of the subdistricts in \( M_A \).

Party B, the second mover, will select a best-response to any proposed set of subdistricts. The strategy \( \sigma_B \) is a mapping from the set of valid groupings of subdistricts to a single map, \( \tilde{M}_B \), such that
\[ \tilde{M}_B \equiv \sigma_B(M_A) \in \arg\max U_B(Q(M_A)). \]

Because voters themselves are indivisible and the districts in this setup consist of sets of voters, any game has a fixed number of possible districts. Also, the second-mover knows what the first mover has chosen to do. In a finite extensive game with perfect information, such as DCP, there exists a subgame perfect equilibrium (Osborne et al., 2004, p. 173). Furthermore, it can be solved using backward induction. For the map \( M_A \) selected by Party A, Party B will examine all of the possible maps it could draw, \( Q(M_A) \). From these possibilities, it will select the map \( \tilde{M}_B \in Q(M_A) \) that maximizes the percentage of seats won by Party B. For every possible map \( M_A \in \mathcal{M} \), Party A can anticipate what ultimate map \( M_B \) Party B would draw. Therefore, it selects the map \( \tilde{M}_A \) that maximizes the percentage of seats won by Party A subject to Party B’s best-response pairings, with payoff
\[ U_A(\sigma_B(\tilde{M}_A)). \]

\[ \footnote{A related game could involve defining \( kN \) subdistricts where \( k \) is a positive integer greater than 2.} \]

\[ \footnote{Valid districts are contiguous (except where required by geographic features such as islands) and have equal population. We do not impose any compactness, split geography, or other restrictions, but such limitations could be included here. The one exception to this is that valid districts may not include “donuts,” where one district entirely encircles another.} \]

\[ \footnote{To rule out a Definer drawing maps with one or fewer valid Combine stage responses, the procedure can also require a Define stage map \( M_A \) with more than one valid Combine state map in \( Q(M_A) \). For evaluation of real-world state and congressional district maps in this paper, we require \( |Q(M_A)| > 10 \). We have found that this restriction rarely binds in practice.} \]

\[ \footnote{Equivalent to minimizing the percentage of seats won by opposing Party B.} \]
This procedure reduces partisan gerrymandering by limiting the efficacy of the most important strategy for gerrymandering—packing the opposing party’s voters into as few noncompetitive districts as possible Friedman and Holden (2008). Consider a state, to be divided into five districts, with voters evenly divided between the two parties. If Republicans can draw one district with 80% Democrats, they could likely draw the four remaining districts to produce Republican wins by a narrow margin. However, this “packing” strategy fails under DCP—if Republicans draw two 80% Democratic subdistricts, then in the combine stage Democrats would refuse to combine the subdistricts and instead pair them with neighboring Republican subdistricts to win the two resulting districts. While specific political geographies can still enable packing in some places, the ability to do so is significantly constrained.

We assess outcomes by measuring the difference in seats won by each party under URP and DCP. We compare redistricting procedures in terms of (1) the advantage, which we will term definer’s advantage, conferred to the map-drawing party by a redistricting procedure and (2) partisan bias induced by a redistricting procedure.¹¹ SI Appendix C describes these measures in detail.

A Simple Example. A simple example illustrates the DCP framework. Consider a (simplified) map of Iowa, as in SI Appendix Figure S1, with 30 equally-populated precincts and an overall statewide vote share of 50% for the Democrats and 50% for the Republicans.¹²

Suppose redistricting requires that the state be divided into five equally populated contiguous districts (which could occur if Iowa were to gain an additional Congressional district due to reapportionment). Given that each of the thirty precincts has the same population, the state must be divided into five districts of six precincts each. Under these assumptions, there are 27,250 possible maps.

If Democrats re-draw the map unilaterally and maximize the number of seats won, they can construct a map where they win four seats and Republicans win one seat. Figure 1(a) shows one such map (out of many equivalent possibilities) drawn by the Democrats unilaterally. Districts won by Democrats (Republicans) are denoted with a blue (red) outline. In this example, Republicans are packed into District 4, and Democrats win majorities in Districts 1, 2, 3, and 5. Conversely, as illustrated in Figure 1(b), when Republicans act unilaterally, the opposite result is possible: Republicans win four seats, and Democrats win one.

We now apply DCP to this simple example. In the first stage, the defining party will draw a map consisting of ten contiguous subdistricts, each with three precincts. There are 7,713 valid divisions of this map of Iowa. In the second stage, the combining party selects contiguous pairs of subdistricts to create the final district map. The number of possible combinations in the second stage varies based on the subdistricts defined in the first stage. In this example, the number of combinations varies from 2 to 20 possibilities.

For any possible proposed map (i.e., for each sub game), the defining party analyzes the resulting combinations and determines the best-response for the combining party. The defining party chooses the map that minimizes the utility that the combining party gets from making the optimal pairing in the sub game. Given the distribution of voters in our running example, Figure 2 presents the results of DCP for this simple example if the Democrats go first, on the left, and if the Republicans go first, on the right. In this example, there are multiple equilibria and we present just one graphically. The defined subdistrict plan selected by the Democrats results in three seats for Democrats and two seats for Republicans. The Republicans cannot choose any other combination of these subdistricts to improve the outcome. Similarly, if the Republicans move first, then in equilibrium the Republicans win three seats and the Democrats win two seats. Thus, DCP reduces the advantage conferred to the map-drawer/definer. Under URP, there is a three-seat (of five total seats) difference in partisan outcomes depending on who controls the process (δ_D = 0.8 − 0.2 = 0.6), while under DCP there is only a one-seat difference depending on who draws the define-stage map (δ_D = 0.6 − 0.4 = 0.2). In the next section, we extend this example by illustrating DCP’s performance across a wide range of actual state Congressional and legislative maps.

Evaluating the Define-Combine Procedure

Simulated State Congressional and Legislative Maps.
We selected eleven states, varying in size and partisanship, and used map-drawing algorithms to generate thousands of possible maps of 2N subdistricts (the define stage). For each generated map, we then simulated hundreds of different pairings of districts to create the final maps (the combine stage). For each map, we calculated the number of seats won by each party, and then identified the maps that would be chosen by each party unilaterally and by each party in sequence under DCP, based on election results from the 2016 presidential election, adjusted by uniform swing to 50% for each party. This process allows us to assess if DCP would in practice reduce the partisan advantage conferred to the redistricting party along with the bias induced by the redistricting process.

¹¹A definition of partisan bias is the difference between seat share and vote share in a state where votes are split evenly between the parties.

¹²We used a 2016 Iowa precinct shape file to generate the map, simplifying by creating thirty precincts with equal population. Vote share in each precinct is based on 2016 Presidential election totals, with a uniform swing applied to each precinct so that the statewide average is 50% for each party. Note that two-party Democratic vote share in Iowa averaged over the 2012 and 2016 presidential elections is 49%, so this scenario does not stray far from reality.
Simulating valid (contiguous, equally populated) districting maps represents a substantial computational challenge, and scholars have recently developed several different algorithms to do so.\textsuperscript{13} We selected the \texttt{GerryChain} package MGGG (2018), which uses Markov chain Monte Carlo simulations to generate maps that theoretically span the full distribution of possible valid maps.\textsuperscript{14} We selected states based on availability of data, using shapefiles and election data from the Metric Geometry Gerrymandering Group (MGGG).\textsuperscript{15}

For each state, we used the \texttt{GerryChain} algorithm to generate 10 independent Markov chains of 100,000 maps of $2N$ subdistricts, where $N$ is the number of districts in that state, resulting in 1,000,000 define-stage maps per state.\textsuperscript{16} We thinned the chain by selecting 1\% of the simulated maps and then generated up to 500 possible combinations of pairs of districts to evaluate.\textsuperscript{17} Using actual election data, we then identified the best map for the Democrats and for the Republicans under URP and under DCP.\textsuperscript{18}

Figure 3 displays the results for each state under the scenario where the two parties evenly split votes in the

\textsuperscript{13}There are many different algorithm approaches to generate redistricting plans Altman and McDonald (2011); Chen and Rodden (2013); Chen and Cottrell (2016); Cho and Liu (2016); MGGG (2018); Magleby and Mosesson (2018); Fifield et al. (2020).

\textsuperscript{14}We also used the \texttt{redist} package (Fifield et al., 2020) to simulate Congressional district maps and produced nearly identical results for each state.

\textsuperscript{15}Precinct shapefiles and election data available from: \url{https://github.com/mggg-states}.

\textsuperscript{16}Maps are drawn and evaluated using precinct-level population data and election results. In each chain we restricted population deviations to 2\% for Congressional Districts and 10\% for State Senate Districts, and used a different set of initial districts to increase the probability of simulating districts across the full possible distribution. Full replication code available upon publication.

\textsuperscript{17}For some of the maps, particularly in states with fewer Congressional districts, there were not 500 possible combinations of districts for every defined map. In these cases we used all possible unique combinations. SI Appendix Tables S2 and S3 summarize the number of maps drawn and sets generated for each state and district type. For a few maps, there were no valid combinations; these maps were discarded. See SI Appendix Figure S13 for an example.

\textsuperscript{18}To be clear, we are reporting the “extreme values”—the most or fewest districts a party would win, given the fixed distribution of voters in a state—rather than an average of all simulated maps.
state. In all eleven states (for both Congressional and state senate districts), there is a sizable gap in seats depending on which party unilaterally draws district maps (e.g., a significant definer’s advantage). For example, in Virginia, we find that Democrats could draw maps where they win eight of eleven Congressional districts and twenty-six of forty state senate districts. In contrast, Republicans could draw maps where Democrats win only three of the eleven Congressional districts and thirteen of the forty state senate districts. Averaging across all eleven states, URP confers an advantage equal to almost half of all Congressional districts ($\delta_D = .48$) and close to a third of all state senate districts ($\delta_D = .29$) when parties seek to maximize seats won.

Under DCP, the gap for Congressional districts is reduced to one or zero seats in every state, except in Maryland, where Democrats have a two-seat advantage, and to one or zero seats in every state for state senate districts. In every state, DCP produces maps with a narrower range of outcomes as compared to URP, regardless of which party defines subdistricts and which party combines them. On average, the definer’s advantage is close to zero ($\delta_D = .07$) for Congressional districts. When drawing state senate districts, the parties receive the same number of seats in four states regardless of which party is the definer. In all but one remaining state, there is a one-seat advantage for the combining party (in North Carolina the combining party retains a two-seat advantage). Overall, $\delta_D = -.016$ for state senate districts, where the negative sign on the latter term indicates a very slight second-mover or combiner’s advantage. The increased numbers of legislative districts yield the opposite of what we found for the less numerous Congressional districts, which exhibited a first-mover or definer’s advantage for every state except Texas (which has 36 Congressional districts). Looking across all states for both Congressional and state senate districts, DCP dramatically restricted the ability of each party to gerrymander as compared to URP in every case.

While DCP substantially reduces the gap in potential outcomes based on which party controls redistricting, the resulting map may still favor a particular party. For example, in Georgia, the best unilateral Democratic map produced 9 Democratic wins out of 14 districts, while the best unilateral Republican map produced 12 Republican wins. Under DCP, the resulting maps (regardless of first-mover), produced five Democratic seats and nine Republican seats, even though both parties receive 50% of the vote. This is due to the geographic distribution of voters across the state (see SI Appendix Figure S14), where Democrats cluster in urban areas.\footnote{Past research has identified how the spatial clustering of one party can lead to an increased likelihood of facing an electoral disadvantage, even when drawing maps randomly (i.e., without an explicit partisan bias) Chen and Rodden (2013).}

Figure 4 illustrates the relationship between geographic bias (x-axis) and seats won under unilateral redistricting or DCP (y-axis). We estimate geographic bias as the percentage of seats won by each party across the full set of randomly generated maps Chen and Rodden (2013). When Democrats win more (less) than 50% of seats averaged across all simulations, it indicates that the state’s political geography inherently favors Democrats (Republicans), even if district maps are not drawn purposefully to favor one party. Iowa (only Congressional districts),
Maryland, and Texas exhibit slight geographic biases favoring Democrats while all other states exhibit geographic biases favoring Republicans. The y-axis of the figure measures the share of Democratic seats won for a specific redistricting procedure in each state. Geographic bias favoring Democrats correlates positively with Democratic seat share under either redistricting procedure.

Critically, however, states with little or no geographic bias (located near the 0.5 line on the x-axis) exhibit considerably less partisan bias under DCP compared to URP. The states with the lowest levels of observed geographic bias—Texas and Virginia—illustrate this point, as seat shares for Democrats hover near 0.5 under DCP (e.g., no partisan bias) but exhibit biases worth 15 percent or more of all Congressional seats for URP.

**Grid Maps.** For state maps and elections, DCP reduces both bias and the advantage conferred to the party controlling redistricting when compared to URP. But real-world examples have just one geographic distribution of voters per state. To test the robustness of DCP and to explore its properties, we use simulated grid maps that allow for (1) different voter distributions with varying degrees of geographic clustering and (2) different levels of state-wide partisanship, (3) varying objectives for the redistricting parties, and (4) varying numbers of districts for a fixed total population. We explore how each of these extensions influence the properties of URP and DCP. Overall, DCP continues to reduce gerrymandering dramatically when varying the degree of state-wide partisanship, the geographic clustering, the parties’ objective functions, and the numbers of districts. We summarize our findings in this section, and provide more detailed accounts of each extension in SI Appendix E.

To simulate grid maps, we define a grid of equal-population precincts, randomly assign partisanship (Party A or Party B) to voters in each precinct while fixing map-wide partisan composition at a specific value, and then solve for the URP and DCP maps that each party would select in redistricting. We report the average across the randomly-generated voter distributions to characterize the results. SI Appendix E.1 describes the simulations in full detail.

**Varying Partisan Composition of Voters.** DCP produces significant improvements in terms of reduced Definer’s Advantage and bias across a range of different state-wide vote shares. Grid simulations show that partisan advantage conferred to a unilateral redistricter ($\delta_U^V$) peaks when the parties evenly split the state-wide vote. In contrast, the partisan advantage under DCP ($\delta_D^V$) remains low across the full distribution of possible state-wide vote shares (see SI Appendix Figure S3). Second, DCP reduces bias due to redistricting most at vote share $V = 0.5$. When the vote share of the unilateral redistrictor or definer is very high (e.g., over 60 percent state-wide), DCP’s ability to...
reduce bias diminishes as the majority party begins to win a large majority of the seats regardless of the redistricting process. SI Appendix E.2 provides the full analysis.

Alternative Objective Functions for Redistricting Parties. Parties may have objectives other than maximizing seats won in the next election. For example, parties that are “running scared” may seek to maximize partisan bias and minimize responsiveness to insulate against future partisan swings. Conversely, parties optimistic about the future may favor plans maximizing responsiveness and minimizing bias (Katz, King, and Rosenblatt, 2020).

SI Appendix E.3 examines these possibilities in detail. When parties hold differing objectives—one party maximizing responsiveness and the other bias—our same core results obtain. DCP reduces the advantage conferred to the redistricting party / definer by almost 70 percent for relatively competitive state-wide vote shares (e.g., between 0.45 and 0.55). Reductions in bias from implementing DCP operate similarly.

Geographic Partisan Clustering. We examine overall geographic clustering of parties (with the parties equally clustered) as well as differential clustering that induces a geographic bias favoring one party. When both parties are more clustered overall, both URP and DCP exhibit larger advantages for the unilateral redistricter/defining party. However, no matter the level of clustering, the magnitude of the advantage is at least 90% less under DCP than URP. Similar results obtain for bias attributable to redistricting.

When geographic bias favors one party only, the favored party wins a higher seat share no matter the redistricting procedure; however, DCP essentially eliminates the advantage conferred to the redistricting party as well as bias attributable to redistricting (see Figure S8). See SI Appendix E.4 for a full discussion.

Number of Districts. The ratio of the population in a state to the number of districts has implications for gerrymandering in general and the effects of URP versus DCP in particular. This ratio is particularly relevant for understanding the properties of redistricting procedures for contexts with a fixed population but different numbers of districts, such as for a state’s Congressional district map versus legislative district map (since most states have more legislative districts than Congressional districts).

As the number of districts increases (for a fixed state-wide population), the advantage conferred to a unilateral redistricter increases up to a threshold number of districts and then decreases, converging to zero. See SI Appendix Figure S9. For DCP, simulations reveal that over a threshold number of districts, the definer’s advantage turns negative (e.g., the second-mover or combiner gains a slight advantage). To see why, consider the case where a state with population \( N \) uses DCP to create \( \frac{N}{2} \) districts. The definer creates \( N \) sub-districts (i.e., each person is a sub-district); then, the combiner’s problem is exactly analogous to the problem of a unilateral redistricter creating \( \frac{N}{2} \) districts—and so the combiner retains the same advantages as would a unilateral redistricter.\(^{21}\)

DCP reduces bias due to redistricting by more than 80% for most numbers of districts. The smallest reductions in bias occur for very small numbers of districts and when the number of districts equals the actual population in the state. SI Appendix E.5 provides an in-depth discussion of these issues.

Implementing the Define-Combine Procedure

How could DCP be implemented in practice?

In states where the legislature controls redistricting, the legislature could adopt DCP in several different ways. Most straightforwardly, if only legislative rules govern the current process, the legislature could change the rules and require that the committee responsible for drawing maps use DCP. Further rule changes might require that a map produced by this process not be amended on the floor or by the other chamber. Commission-based states could incorporate DCP as a guiding tool in their consideration of potential maps, or a legislature could pass a law requiring its use. States considering an independent commission but wary of ceding control of the map-drawing process to an independent member of the commission might instead create a commission with an even number of partisans and require the use of DCP instead.

Perhaps most plausibly, courts could use this process when ordering remedial maps—rather than ordering the state legislature to draw a new map under specific guidelines, or selecting a special master to draw a new map. For example, in Common Cause v. Lewis, the court ordered that the state legislature draw remedial maps in a non-partisan manner. DCP would serve as a simple and efficient framework for future cases like this one. State court judges might also utilize DCP to better understand what “fair” redistricting outcomes look like in their state, using resulting maps as one piece of evidence in partisan gerrymandering cases.

Another key issue for states to adjudicate will be who participates in the “Define” stage and who participates in the “Combine” stage. States could assign the order randomly, alternate between parties each districting cycle, or employ partisan factors, such as the majority party in the legislature or the party of the governor. If the majority party currently enjoys a procedural advantage in the

\(^{21}\)Along these lines, DCP could be modified to involve combining 3 or more subdistricts into a single district in order to alter the advantages conferred to the first and second mover. Holding everything else constant, as the number of subdistricts combined increases, the advantage of the second mover will be weakly increasing. In small states, such an approach could be used to eliminate a first-mover advantage.
redistricting process they may be more willing to accept DCP as a reform if they can choose the role that will still offer them an edge, even though adopting DCP will diminish their advantage—especially if they are worried about future electoral prospects or a court-ordered intervention. Importantly, since partisan outcomes converge under DCP, the stakes over determining the first mover are lower than under unilateral redistricting.

DCP provides the flexibility to be integrated with other approaches to redistricting and to address concerns other than partisan gerrymandering. To address communities of interest, redistricters could apply DCP after freezing a district or districts in place to ensure a map retains specific, desirable properties. For example, if future redistricters wanted to maintain an existing majority-minority district, then the parties could freeze the district in place and use DCP for the remaining area in the state. Similarly, DCP could also be applied in conjunction with other redistricting proposals, such as dividing a state in two and allowing each party to serve as Definer in one part of the state.

Conclusion

DCP features simple rules, clear strategies for each party, and an efficient implementation framework for small and large numbers of districts alike. One obstacle to implementation for past theoretical approaches has been the difficulty for decision makers in either party to predict outcomes in the real world. DCP, as we have demonstrated, can be applied to real-world geographic data, which allows analysts to predict outcomes and reduce uncertainty surrounding the redistricting process. Furthermore, DCP significantly reduces the advantages conferred to the redistricting party and results in maps more likely to reflect the will of voters. These advantages hold up across a variety of different contexts reflecting the political and geographic heterogeneity of the states.

There are many challenges in using automated algorithms to aid in the redistricting process (Cho and Cain, 2020). In some cases, advances in computing power and the ability of politicians to consider a large range of maps exacerbates partisan gerrymandering, rather than alleviating it, as partisan mapmakers use map-drawing algorithms to devise increasingly gerrymandered maps. DCP provides an approach that utilizes advances in computing to produce less biased maps—ones where the process-based algorithm itself constrains partisan motives. Additionally, the transparency inherent in DCP will allow for more open deliberation over maps proposed in redistricting sessions, subjecting them to “increased scrutiny” by the public.

Political parties in power will always oppose ceding it, but this is doubly so when the choices they face require embracing significant uncertainty about future political outcomes. Because DCP is a two-stage game and existing computing resources can help solve it, our proposal represents a step towards providing an alternative mechanism to legislature-based or commission-based redistricting that is actually feasible to implement. At the same time, the framework gives parties the autonomy to respect communities of interest, geographic boundaries, and other political concerns—that is, to internalize the wide range of factors that play important roles in decisions about redistricting—while nonetheless tempering the partisan bias that tends to emerge during redistricting. By involving both parties but setting them in opposition to each other, rather than requiring bipartisan cooperation or independent third-party mediators, DCP offers a partisan solution to the extraordinarily partisan process that is redistricting.

References


A. Limitations of Current Partisan Gerrymandering Fixes

We can divide current solutions to partisan gerrymandering into two classes. First, there are solutions that are currently used in at least one of the fifty states. These include, for example, legislature-involved redistricting commissions, independent redistricting commissions, and judicial intervention to reduce partisan gerrymandering. Second, there are proposals—generally put forth by researchers—that move beyond currently implemented solutions and lay out some other mechanism by which maps are drawn. These include methods where the parties draw districts by alternating back and forth; many of these approaches are inspired by the cake-cutting problem and principles of fair division (i.e., how to divide a good, such as a cake, fairly between two parties) (Brams and Taylor, 1996).

A.1. Already-Implemented Solutions. Citizens have expressed deep dissatisfaction with redistricting procedures currently adopted in the states. For example, fewer than 25% of respondents in the Cooperative Congressional Election Survey answered affirmatively when asked whether redistricting in their state was fair (Schaffner and Ansolabehere, 2015). In a number of states, voters or legislators have responded by establishing redistricting commissions that are meant to de-politicize the map-making process and produce districts that are more fair.

Table S1 reports the specific type of commission used in each commission-based state. In total, 29 states draw their Congressional district maps through the legislature exclusively, while the rest use some sort of redistricting commission. The redistricting process in the majority of commission-based states, however, is not in fact independent from the state legislature. Of the 21 states that use some form of redistricting commissions, only nine states have truly independent commissions that are able to create maps without input or approval of the state legislature. Advisory commissions assist the legislature as it draws district boundaries, but the legislature approves the maps. Political or politician commissions are mainly comprised of elected officials. In backup-commission states, a commission, sometimes comprised of politicians or politician-appointed members, only plays a role if the legislature fails to pass a districting plan within a certain time period. Finally, independent commissions are distinct from the others as they do not include public officials or legislators.

All told, 12 of the 21 commission-based states do not have a redistricting process for their Congressional districts that is meaningfully independent from the state legislature; as a result, the map-drawing process remains subject to the same partisan pressures as in states with legislature-drawn maps. Two of the nine states with truly independent commissions, Alaska and Montana, use their commissions to draw state legislative districts but since they only have one Congressional district their commissions do not actually engage in Congressional redistricting. Of the remaining states that have established independent redistricting commissions for redrawing their Congressional districts, nearly all rely on a member or members to act neutrally (often these members are not affiliated with either of the two major political parties). The logic behind this design is that the two parties will have to appeal to a neutral arbiter—the independent member(s) of the commission—in order to achieve a majority and pass a map. In theory, this could cause both parties to curb their partisan gerrymandering efforts in order to create a fairer map appealing to a neutral (and presumably more moderate) commission member.

Scholars have not reached a consensus on the benefits of independent commissions. One study examines the efficacy of redistricting commissions in seven Western states and compares them to five non-commission states in the West, and finds that redistricting commissions do not out-perform legislatures when judged by the metric of drawing compact, competitive districts that preserve preexisting political boundaries Miller and Grofman (2013). (On the other hand, the same authors find that commissions seem to excel at producing maps “on time” that avoid litigation.) Others have found that there are more competitive districts in commission-drawn maps in the 1990s and 2000s redistricting.

22Though commission-based states did register significantly higher approval rates than legislature-based states.
23We use classifications from Justin Levitt’s website, All About Redistricting: Who Draws the Lines?, with some additional classification Levitt (2020).
24Four of these 29 states have only have one Congressional districts and do not actually engage in Congressional redistricting.
25Selection methods for independent commissions vary significantly—in Arizona, Idaho, Montana, and Washington majority and minority party leaders appoint commissioners, while judges make appointment decisions in Colorado. Alaska has two members chosen by the governor, two by party leaders in the state legislature, and the last by the state supreme court chief justice. California has a process that involves narrowing a pool of applicants down, randomly selecting some members, and then having those members choose the remaining members. Utah’s new independent commission will have all members chosen by state legislators, with the governor choosing the commission chair, though all commission members must not be affiliated with any political party nor have voted in any political party’s primary elections in the past five years. In the case of Michigan’s new independent commission for the 2020 redistricting cycle, independent commissions members were selected randomly from a pool of qualified applicants.
26Only Idaho (6 members) and Washington (4 members and 1 non-voting member) have perfectly balanced (by partisanship) independent commissions, and in these cases some bipartisan cooperation is needed for them to successfully create district maps, though researchers have illustrated that, in practice, balanced commissions may produce incumbent-protecting gerrymanders (McDonald, 2004).
cycles Carson and Crespin (2004), and that independent commissions are more likely to follow traditional redistricting principles, including compactness, splitting fewer political subdivisions, and preserving the cores of existing districts Edwards et al. (2017). Recent research using simulations to consider a set of alternative maps that could have been enacted by independent commissions finds that independent commissions insulate incumbent legislators to the same degree that party-controlled legislative redistricting does, suggesting that independent commissions may not be as neutral as many suppose Henderson, Hamel, and Goldzimer (2018).

The effectiveness of independent commissions also hinges crucially on who staffs them. A Brennan Center report notes that “the strength and independence of the [commissioner] selection process was, by far, the most important determinant of a commission’s success” (Redistricting Commissions: What Works, 2018). Even with an independent staffing process, however, independent commissions do not quell the partisan anger over redistricting controversies. Those who have studied independent commissions note that “the decisions of such commissions may generate partisan rancor comparable to what we see from states where one party entirely controls the redistricting process and engages in a partisan gerrymander” (Miller and Grofman, 2013, p. 648), and that “[o]ften, commissioners have strong common prior beliefs about the likely partisanship of the tiebreaker, and therefore balk at compromise during initial negotiations. Once chosen, the tiebreaker then sides with one of the parties and a partisan plan is adopted” (McDonald, 2004, p. 383). Similarly, the Brennan Center report notes that “states that used a tiebreaker model popular in earlier reforms experienced much lower levels of satisfaction, mainly because the tiebreaker tended to end up siding with one party or the other, resulting in a winner-take-all effect” (Redistricting Commissions: What Works, 2018).

Last of all, the establishment of an independent redistricting commission is not a realistic options for many citizens. According to the National Conference of State Legislatures, slightly more than half of U.S. states do not have a legislative process allowing statutes or state constitutional amendments by initiative. Of the 24 states that do, nine have independent redistricting commissions already. The states with the most intense partisan gerrymandering do not have an initiative process, and legislatures in those states are also very unlikely to voluntarily relinquish authority over redistricting to an independent commission. For example in Maryland, North Carolina, Pennsylvania, Texas, Virginia, and Wisconsin, voters cannot feasibly establish non-partisan independent redistricting commissions since these states do not have an initiative process, and the legislatures seem unlikely to give up their redistricting power.

These problems with the creation and effectiveness of commissions show that independent redistricting commissions do not offer a silver-bullet solution to partisan gerrymandering in most states. Regardless of who draws the lines, many states have instead looked to the courts for relief from partisan gerrymandering. Existing legal remedies, however, have met with several obstacles.

One of the largest obstacles to effective judicial intervention is that courts lack effective guidelines and standards to adjudicate partisan gerrymandering litigation. At a minimum, courts must decide (1) how to measure and evaluate partisan gerrymandering, (2) how to compare multiple maps, and (3) at what threshold there is too much partisan gerrymandering. But none of these three issues have been settled. Any solution needs to cut through the “sociological gobbledygook” in a way perceived as non-partisan and legally sound (quoting Chief Justice Roberts during Oral Arguments for Gill v. Whitford, October 3, 2017). Additionally, the Supreme Court’s decision Rucho v. Common Cause (2019) effectively barred the federal judicial from future intervention in partisan gerrymandering litigation. This has left state courts to adjudicate partisan gerrymandering claims, and relegates the judicial intervention option to a much less effective state-by-state approach.

State Supreme Courts have recently struck down redistricting plans for being unconstitutional partisan gerrymanders (according to state law). In Florida, the courts based their decision in League of Women Voters v. Detzner (2015) on a “Fair Districts” amendment prohibiting partisan gerrymandering, which voters had previously added to the state constitution through a popular initiative. In both Pennsylvania (League of Women Voters v. Commonwealth

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28. There exists no legal consensus on how to best identify instances of partisan gerrymandering, despite a plethora of new partisan gerrymandering metrics developed in the past few decades. Since the Supreme Court’s decision in Vieth v. Jubelirer (2004), finding a standard to judge partisan gerrymandering has remained a challenge. Measures like the Efficiency Gap (Stephanopoulos and McGhee, 2015), the Mean-Median Difference (McDonald and Best, 2015), and Partisan Fairness (King and Browning, 1987; Grofman and King, 2007) have grown increasingly common, but courts have not settled on one. Each approach has some mix of desirable and undesirable features (Stephanopoulos and McGhee, 2018).

29. Courts sometimes rely on simulated or counterfactual election results in order to create a distribution of possible maps against which the actual or proposed redistricting plans can be compared. Current computational limitations make it impossible to create the full distribution of possible maps, so simulations rely on creating a representative sample of possible maps as a baseline (Cho and Liu, 2016). Experts continue to debate whether particular simulation methods create a “true” distribution of possible maps, and the courts must navigate among competing methods, Cirincione, Darling, and O’Rourke (2000); Altman and McDonald (2011); Chen and Rodden (2013); Chen and Cottrell (2016); Cho and Liu (2016); Magleby and Moessner (2018); Duchin (2018); Fifield et al. (2020).

of Pennsylvania, 2018) and North Carolina (League of Women Voters v. Rucho, 2018; reconsidered and reaffirmed 2019), the courts relied on more generic language in the state constitutions ensuring “free elections.”31 Only some states, however, have existing state laws or constitutional amendments that provide a legal basis to limit partisan gerrymandering. For example, while 30 states have some version of “free election” clauses in their constitutions, only 18 also require “equal” or “open” elections.32 Notably, of 41 the states that do not have an independent redistricting commission, 18 of them have neither a “free election” clause nor an “open” or “equal” provision in their state constitutions.

Overall, the existing attempts to fix partisan gerrymandering have resulted in a patchwork of solutions with highly limited effectiveness. Many voters live in states that cannot feasibly implement the commission-based solutions that have had success in other states, and judicial intervention is limited to state courts which often do not have constitutional provisions that allow them to reduce partisan gerrymandering. And lack of citizen-led initiative procedures in many states makes it impossible for voters to solve this issue without the help and approval of their partisan state legislators. This insufficient patchwork leaves citizens with little recourse to address the degradation of representation in their states caused by partisan gerrymandering.

A.2. Other Proposed Solutions. Some of the most promising alternative solutions to gerrymandering draw inspiration from the cake-cutting problem; how do two people perform the fair division of a piece of cake without the need of third-party intervention? The solution is to arbitrarily choose one as the first mover; she divides the cake and then the second-mover may choose between either of the pieces. This logic, applied to geography, has inspired several redistricting proposals.

One proposal is to have an independent third party divide the state into two and then each party negotiates over who gets to redistrict one section of the state Landau, Reid, and Yershov (2009). The parties each independently redistrict their agreed-upon parts of the state. Combining the two results in a final map. In another proposal, each of two parties alternate back and forth drawing district maps Pegden, Procaccia, and Yu (2017). Termed “I-cut-you-freeze,” the protocol involves a back and forth where one party draws a map, the other party freezes in place one district from that map, and then redraws a new district map for the remaining area in the state. The players alternate between “cutting” and “freezing” until producing a full map.

Neither of these approaches has seen any take-up in the real world. The difficulties of implementing these solutions in practice are several-fold. In the first proposal, the process requires a neutral third party to take the initial step of dividing the state into two parts, which has proven to be a stumbling block in the past Landau, Reid, and Yershov (2009). Both approaches abstract from real-world geographies and do not place constraints on how voters are assigned to districts Landau, Reid, and Yershov (2009); Pegden, Procaccia, and Yu (2017). Furthermore, because they involve multiple stages of bargaining between the parties, these approaches are impractical to simulate in real-world contexts using actual geographies and voter rolls. Thus, lack of information about implementation and potential results with real electoral geography and population information make it unlikely that decision makers would adopt these protocols.

Other researchers have proposed a protocol with a similar “I-cut-you-freeze” style, but with an explicitly spatial addition to the process Ely (2019). The first party draws a full set of districts. Any district that is convex is locked into place. However, the second party has the ability to redraw any non-convex districts so that they are convex. This two-stage process assures the creation of a map without misshapen districts. However, this proposal also meets with some practical issues. First, in some states it is likely not possible to meet equal population requirements while also maintaining convex districts. Second, even states with convex districts can be extraordinarily biased in favor of one party, depending on the geographical distribution of voters (Alexeev and Mixon, 2017). A final proposal involves a method that divides the state in two and allows each party to redistrict their half, with the additional constraint that each party draws a share of districts roughly proportional to the party’s statewide vote share in the last Congressional election Brams (2020). In essence, this method seeks to let the parties create their fair share of gerrymandered districts.

All of the proposed solutions involve either a third-party neutral arbiter, are difficult to implement in practice, or have uncertain outcomes that are hard if not impossible to predict computationally. Our Define-Combine Procedure is designed to address these difficulties. Unlike the already-implemented fixes or other proposed solutions, DCP does not require an independent third party to ensure that districts are fair, and it is possible to predict the outcomes

31 For North Carolina, the courts concluded that the redistricting process was not consistent with a broad reading of Section 10 of the North Carolina State Constitution, which states that “All elections shall be free.” Similarly in Pennsylvania, the courts found that the challenged map violated the “Free and Equal Elections” Clause (Article 1, Section 5) of the Pennsylvania State Constitution.
33 For a district to be convex, a straight line can be drawn between any two points in the district and all of the line remains inside the district.
of DCP using simulations. An additional benefit is that DCP could be combined with many existing solutions or proposals - for example, by having an existing redistricting commission use the DCP to create legislative maps for a state, or by first freezing certain districts and then using DCP on the rest to produce a final map. This represents a substantial step towards implementing a process-based solution to the problem of partisan gerrymandering.
B. Current Redistricting Procedures

Table S1. Redistricting Procedures by State, for U.S. House Districts

<table>
<thead>
<tr>
<th>Legislature Only (29)</th>
<th>Legislature-Involving Commissions</th>
<th>Independent Commissions (9)</th>
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<tr>
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<td>Advisory (6)</td>
<td>Political (4)</td>
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<td>Alabama</td>
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<td>Arkansas</td>
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<td>Pennsylvania</td>
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<td>Delaware*</td>
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<td>Florida</td>
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<tr>
<td>Louisiana</td>
<td>North Dakota*</td>
<td>Wyoming*</td>
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<td>Maryland</td>
<td>Oklahoma</td>
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Advisory Commission: Assists the legislature in drawing the maps, but the legislature has the ultimate power to approve or alter the final district maps; Political Commission: Legislation as a whole isn’t officially involved, but the members of the commission are politicians or elected officials; Backup Commission: Step in if the legislature does not pass a districting plan by a certain deadline—these vary in their composition and procedures as well, but are almost always comprised of politicians (governor, secretary of state, state legislators, or members selected by political leadership); Independent Commission: Commissions that have no politicians or elected officials on them, and whose maps are not subjective to legislature approval.

1 In the 2020 redistricting cycle, Ohio will have a seven-person politician commission that draws lines if the legislature does not create a map with three-fifths legislature support. The commission’s map must have the support of two minority party legislators, who are required to be on the commission. If both the legislature and politician commission fail to enact a map, the majority party can adopt a map without minority support that would last for four years. https://redistricting.lls.edu/state/ohio/

2 For the 2020 redistricting cycle, Utah will have a seven-member independent commission to draw their state legislative and U.S. House districts. While the members cannot be elected officials and are required to be unaffiliated with any political party, they are ultimately appointed by elected officials (Proposition 4, “Independent Advisory Commission on Redistricting Initiative”, passed November 6, 2018: https://elections.utah.gov/Media/Default/2018%20Election/Issues%20on%20the%20Ballot/Proposal%204-%20-%20Full%20Text.pdf).

*State only has one U.S. House district; state legislative redistricting authority used for classification.

C. Measurement of Redistricting Plans

C.1. Comparing Plans. Consider an electoral system with seats-votes function $S_M(\nu_1, \ldots, \nu_N)$ for a map $M$, which takes as an input district-level vote shares $\nu_1, \ldots, \nu_N$ and yields as an output a seat share. The state-wide vote share $V$ is the average of district-level vote shares (importantly, elections with identical state-wide average vote share $V$ but different realizations of $\nu_1, \ldots, \nu_N$ could result in different winning candidates). Conditioning on a state-wide vote share $V$, we can find the average seat share by taking the expected value of the function, over the joint distribution for $\nu_1, \ldots, \nu_N$—e.g., $E(S_M(\nu_1, \ldots, \nu_N) \mid V) = S_M(V)$. Note that $S_M(0.5) \neq 0.5$ indicates an electoral system with partisan bias, which could be due to inherent geographic bias (Chen and Rodden, 2013), gerrymandering, or both.

The definer’s advantage depends on how the seat share changes when Party A unilaterally redistricts compared to when Party B unilaterally redistricts, $\delta^U_A = S_M^A(0.5) - S_M^B(0.5)$. Similarly, $\delta^D_A = S_M^A(0.5) - S_M^D(0.5)$ indicates the definer’s advantage under DCP. A large positive value for $\delta^U_A$ indicates that the party controlling the URP can reap a significant electoral advantage through gerrymandering; a large positive value for $\delta^D_A$ indicates that the definer or first-mover in DCP can reap a significant electoral advantage. A negative value indicates a second-mover advantage or definer’s advantage. Redistricting procedures that minimize the absolute value of this quantity tend towards providing both parties equal treatment.

Second, to determine how a redistricting procedure affects bias, we directly compare seat shares for each procedure when the two parties evenly split votes, while accounting for geographic bias due to the spatial distribution of voters. If $|S_M^A(0.5) - \gamma| > |S_M^B(0.5) - \gamma|$ (where $M_A$ denotes the optimal map for Party A from unilateral redistricting,

$34$ Notation used here is similar to Katz, King, and Rosenblatt (Katz, King, and Rosenblatt, 2020).
the optimal map for Party A from DCP, and $\gamma = 0.5 + \beta_g$ adjusts for geographic bias present in the state ($\beta_g$), then DCP reduces bias due to redistricting as compared to URP.

Partisan gerrymandering may pose a problem for an electoral system if there exist large differences in seats won depending on which party controls the redistricting process. Consider a state with unilateral redistricting and vote share $V = 0.5$; suppose Party A wins 75% of the seats if it draws the map whereas Party B wins 70% of the seats if it draws the map. Such a map appears to confer a large partisan advantage to whichever party controls redistricting; 45% of seats in the legislature change hands depending on the party that draws the map. Alternatively, suppose that Party A wins 52% of the seats if it draws the map, and Party B wins 50% of the seats if it draws the map. In this case, partisan gerrymandering represents a smaller problem, with a swing of 2 percentage points depending on the party controlling the process.

D. Iowa Example

![A simplified map of Iowa with 30 equally populated precincts. Dark red (blue) precincts denote higher Republican (Democratic) vote shares.](image)

E. Grid Maps

We now extend the analysis by simulating thousands of different distributions of voters on a grid map and then identifying the maps selected by each party under unilateral redistricting and under DCP.

E.1. Grid Map Simulations. The simulations proceed in four steps:

1. Define a grid of $P$ precincts; each will have the same population.

2. Generate a random distribution of voters in each precinct. Instead of making each precinct either one Party A voter or one Party B voter, each precinct contains the same population size, but with a randomly selected percentage of voters supporting each party. First, we pick a target vote share $m$ for Party A in the grid as a whole. We vary this across simulations in 2.5% increments from 30% to 70%. For each target vote share, we draw a vote share for each precinct from a truncated normal distribution with mean $m$.\[35\] We repeat this process 100 times for each level of $m$, resulting in 1,700 different distributions of voters.

3. Generate potential maps for the grid:

   (a) Generate a set of possible maps of $N$ districts, and a set of possible maps of $2N$ districts. For the simple 30-unit grid, we generated every possible map. For more complex grids, we generated a random sample of maps.

\[35\] The truncated normal distribution is bounded at 0 and 1 and has a standard deviation of 0.25.
Fig. S2. Results for Voter Distribution Simulations on a Simple Grid. Each simulation uses a 5x6 grid.

(b) For the set of $2N$ districts, generate all possible plans that combine pairs of contiguous districts. For the simple grid, we generated every possible combination, and for more complex grids we generated a random sample of combinations.

4. For each distribution of voters, examine the set of generated maps to identify:
   
   (a) The best map for Party A, if Party A chooses a map unilaterally.
   
   (b) The best map for Party B, if Party B chooses a map unilaterally.
   
   (c) The map Party A would choose if it goes first under the Define-Combine Procedure.
   
   (d) The map Party B would choose if it goes first under the Define-Combine Procedure.

   For each identified map we calculate the number of seats won by each party.

E.2. Varying Partisan Composition of Voters . Figure S2 presents the results for a 30-voter grid (5 districts of 6 voters each). The x-axis corresponds to the share of voters supporting Party A, and the y-axis to the share of seats won by Party A. The dotted lines illustrate the average across simulations when Party A (in blue) and Party B (in red) draw the district maps unilaterally. The solid lines display the averages for DCP, with the blue (red) line corresponding to Party A (B) as the definer followed by Party B (A) as the combiner. When one party dominates state-wide vote share (e.g., more than 75% of vote), the results remain similar no matter who controls the unilateral process and no matter if DCP is implemented. However, when both parties are competitive in terms of vote share, significant differences emerge. At $V = 0.5$, the advantage conferred by drawing maps unilaterally is $\delta_U \equiv 0.8 - 0.2 = 0.6$, or three seats. Under DCP the first-mover advantage is $\delta_U \equiv 0.6 - 0.4 = 0.2$, or one seat. All told, implementing DCP on a map evenly divided between the parties will reduce the advantage of the redistricters, as compared to their opponents, by a seat share of 0.4 or 2 seats. DCP does offer the defining party a first-mover advantage in this context of one additional seat over the combiner.

The definer’s advantage in DCP declines as the size of the grid and/or the number of districts increases. Figure S3 displays the average number of seats won for a larger hexagonal grid where 150 precincts are divided into 15 districts. While there is still a substantial gap in seat share when parties draw maps unilaterally, the seat shares under DCP converge, no matter who moves first. For example, when voters in the 150 precinct grid are split evenly between the
parties and there is unilateral redistricting, the party drawing the map gets a seat premium roughly equal to half of all seats on average ($\delta_U^{V} = 0.54$). In contrast, under DCP the seat share remains nearly the same no matter which party goes first ($\delta_D^{V} = 0.07$).

Figure S4 plots the values of $\delta_U^{V}$ and $\delta_D^{V}$ against vote share for the 150 precinct grid. Partisan advantage due to unilateral redistricting ($\delta_U^{V}$) peaks when the state is evenly split. In contrast, the partisan advantage under DCP ($\delta_D^{V}$) remains relatively low across the full distribution. While it increases slightly for vote shares in the range 0.37-0.45 and 0.55-0.63, the defining party still receives an advantage of less than one of sixteen seats, compared to an advantage equal to four of sixteen seats under unilateral redistricting. Across all vote shares, DCP reduces the partisan advantage of going first and substantially limits the ability of each party to gerrymander.

Because we randomly generate these grid maps, the average geographic bias due to clustering of partisans is zero for any vote share. As a result, we may identify bias due to redistricting by a direct examination of the seat shares for unilateral redistricting and for DCP. For both maps simulated in this section, DCP reduces bias most when votes are evenly split; as a party’s vote share increases, the gap in biases due to redistricting procedure narrow, until converging for vote shares 0.6 and above.

The figures above present average numbers of wins for each party or the average values of $\delta_U$ and $\delta_D$, across 1,700 different random distributions of voters across the grids (100 distributions for each mean level of vote share for Party A, from 30% to 70% in increments of 2.5%). However, we also would like to know how DCP performs not just on average but for every possible map. That is, are there any distributions of voters for which DCP does not represent a meaningful improvement over the unilateral case? To address this question, we examine the results from each separate
distribution of voters. For each voter distribution on the 150 precinct grid, we calculated \( \delta^U \), \( \delta^D \), and the difference between them. If \( \delta^D = \delta^U \), then, for that particular voter distribution, DCP fails to improve the outcome. Figure S5 presents scatter plots showing, for each mean level of Party A vote share, the values of \( \delta^U \) (on the x-axis) and \( \delta^D \) (on the y-axis). Points are sized by the number of times that result is realized in each of the 100 simulations for each vote share level. In all cases, \( \delta^D < \delta^U \). In other words, DCP improved the outcome in all 1,100 cases by a meaningful margin.

Fig. S5. Values of \( \delta^U \) and \( \delta^D \) for each simulated voter distribution on the 150 precinct grid.
E.3. Alternative Objectives to Maximizing Seats Won in Next Election. In the simulations above, both parties sought to maximize seats won. However, parties may have objectives other than maximizing current seat share. These differing objectives could include maximizing the odds of maintaining a majority, or preventing losses in future elections rather than just the impending one.

Katz, King, and Rosenblatt (2020) notes that if a majority party controls the redistricting process but is worried about the future (i.e., “running scared”), then the party may favor plans that maximize partisan bias and minimize responsiveness to insulate the party from future partisan swings. In contrast, if a party expects to win a majority of the vote in future elections, they may seek to create districts that are microcosms of the state as a whole by maximizing responsiveness and minimizing bias (Katz, King, and Rosenblatt, 2020). Further complicating matters, the objectives of competing parties could also differ, since a party seeking to maintain a partisan advantage will likely have goals that differ from the party seeking to gain one.

To explore the implications of asymmetric utility functions, we simulated redistricting games where the competing parties had differing objectives: Party A is confident about the future and maximizes responsiveness while Party B is running scared and seeks to maximize bias. Figure S6 illustrates results when Party A earns a vote share over 0.5. The key point revealed by this exercise is that the main findings we obtained when both parties maximized current seats won still hold up. DCP reduces the level of bias as well as the definer’s advantage for state-wide vote shares between roughly 0.5 and 0.65. For vote shares above 0.65, the underlying advantage of Party A is so great that the redistricting procedure no longer matters and Party A simply wins all the seats.

E.4. Geographic Partisan Clustering. Geographic clustering of partisans affects the ability of parties to gerrymander. The clustering of Democratic voters in cities can lead to “unintentional gerrymandering”—even district maps drawn with no intention to gerrymander, such as randomly generated maps, still exhibit partisan biases disadvantaging Democrats due to partisan differences in geographic concentration of voters Chen and Rodden (2013). High levels of clustering disadvantage the clustered party since its votes are more likely to be inefficiently grouped together, leading to more “wasted” votes (Stephanopoulos and McGhee, 2015). Similar arguments may apply to racial gerrymandering, given differential levels of geographic clustering of racial groups (Magleby and Mosesson, 2018). DCP’s effectiveness at reducing partisan gerrymandering, along with the size of the advantage conferred to the first versus second mover, might similarly depend in part on the level of geographic clustering. We explore that possibility here.

We examine two different elements of clustering: (1) overall clustering of partisans, and (2) differential clustering of one political party.

To examine overall clustering, we employ Moran’s I, a common measure of spatial autocorrelation (Moran, 1948). Party A maximizes utility by choosing the option that provides the maximum level of responsiveness. When multiple plans produce the same value of responsiveness, Party A uses wins as tiebreaker. Similarly, Party B maximizes bias and uses wins as a tiebreaker. Katz, King, and Rosenblatt (2020) finds a tradeoff between bias and wins, such that maximizing one should generally reduce the other as well.
Moran’s I Measure

Seats Won by Party A

Vote Share = 0.5, 0−1 draws

Fig. S7. Results by Clustering. Vote Shares = 50%

used in academic political science research and legal work on redistricting (Mayer, 2016). Moran’s I ranges from −1 to 1, with more positive values denoting increased clustering. For example, a 5-by-6 grid with all Party A voters located on the left and all Party B voters on the right yields a Moran’s I of 0.796; a grid with each party’s voters distributed evenly yields a Moran’s I of −1 (see Figure S10). A Moran’s I of 0 indicates randomly dispersed voters with neither clustering nor a pattern of “even” dispersion. SI Appendix F illustrates maps with varying levels of Moran’s I values and provides additional details on Moran’s I calculations.

We first randomly assign single voters to 5-by-6 grids of precincts, setting the probability that a voter supports Party A equal to 50%. For each randomly-drawn grid, we evaluate map-drawing under both unilateral redistricting and DCP, and we also calculate Moran’s I for each grid map. Figure S7 plots the relationship between seats won by Party A and the amount of geographic partisan clustering when Party A support is fixed at 50%.

As clustering increases, the definer’s advantage ($\delta_U$) for unilateral redistricting increases slightly as well. Going from the lowest observed level of Moran’s I to the highest increases $\delta_U$ by about 0.2. How does DCP perform under different levels of geographic partisan clustering? When clustering is low, DCP removes the definer/first-mover advantage (e.g., $\delta_D = 0$). When clustering is high, even under DCP there remains a small but significant definer/first-mover advantage. Nonetheless, DCP’s performance varies only slightly due to clustering in this example, and at all levels of clustering DCP dramatically reduces the advantage of the definer compared maps drawn under unilateral redistricting. Even at the highest levels of observed clustering, DCP eliminates at least 80% of the definer’s advantage observed in unilateral redistricting (e.g., $1 - \delta_D$). Overall, then, map-wide clustering does not appear to meaningfully alter our general results for the definer’s advantage.

Another way to examine the effect of geographic bias is to compare the partisan advantage for one party from the full sample of simulated maps to the results from choosing a map using URP or DCP, as we do for the simulated maps in Figure 4. In our grid simulations, we simulate 100 random draws of voters for each vote share level. We can use the 100 independent draws when vote shares are perfectly split between the parties to calculate the average wins across all maps for each vote draw, and we can compare it to the selected maps for each vote draw. Figure S8 plots the results. Across the 100 draws, geographic bias (the x-axis) varies by about 5% in favor of each party. Geographic bias favoring Party A correlates positively with Party A seat share under either redistricting procedure. As geographic


38 Figures S12 illustrate the relationship between seats won and clustering for different values of state-wide vote share $V$.

39 In addition, note that the range of clustering in our grid examples does not span the full range of possible values of Moran’s I. We do not observe many maps that have negative Moran’s I values simply because they are highly unlikely to occur by our random sampling procedure. (See SI Appendix F for more details.) This is not a limitation of our analysis of the effects of clustering on gerrymandering or DCP for two reasons: first, in a case where partisans are very evenly dispersed (Moran’s I values close to −1), it becomes impossible to gerrymander because there is no way to “crack” or “pack” partisans together, so bias in these maps (under either unilateral redistricting or DCP) will be close to 0. Second, when using real data on partisanship to calculate measures of the dispersion of partisans, all states show significant levels of partisan geographic clustering.
bias decreases, we see less variation in outcomes, and expect both parties to evenly divide the seats.

![Graph](image)

*Fig. S8. Relationship Between Geographic Bias and Outcomes for Simulated Grids*
E.5. Number of Districts and Definer’s Advantage. To explore the relationship between number of districts and the advantage conferred by controlling the redistricting process (e.g., as the one drawing the map under unilateral redistricting or as the Definer under DCP), we simulated potential maps for a hexagonal grid. For all maps, we had a fixed population (4000 hexagons or precincts), fixed vote shares (evenly split between the parties), no systematic voter clustering, and varying numbers of districts to be drawn. The number of districts ranged between 4 and 200. As a result, maps with fewer districts exhibit a higher population to district ratio while maps with more districts had a lower population to district ratio.

As Figure S9 illustrates, unilateral redistricting exhibits a hump shape in terms of the advantage conferred to the party controlling the redistricting process. This advantage peaks for a state with eight districts and then declines. In contrast, under DCP the definer’s advantage is highest for small numbers of districts, turns slightly negative (i.e., indicating a combiner’s advantage / second-mover advantage) and then converges towards zero.

Fig. S9. Relationship Between Number of Districts and First- and Second-Mover Advantage. We generated a 63x64 grid of 4,000 hexagons, each with equal population, and used GerryChain to generate potential maps with different numbers of districts. We generated sets of 10,000 maps each, with 8, 10, 16, 20, 40, 50, 80, 100, 200, 250, and 400 subdistricts, resulting in 4, 5, 8, 10, 20, 25, 40, 50, 100, 125, and 200 final districts. We generated 25 different voter distributions, each with mean of 50% for each party, and calculated the maps selected by each party under the four methods...
These results raise several questions. First, why does the advantage conferred to the unilateral redistrictor appear to be maximized at a relatively low (but not at the lowest) number of districts? To see why, first recall that voters split the vote evenly between both parties. As a result, at least one district will always need to register a majority for the redistricting party’s opponent. This leads to optimal gerrymandering where a party packs the opponent into as few districts as possible. For example, in a map with 4 districts and an evenly split vote, the redistrictor packs half of the opposing party voters into one district and distributes the remaining half equally into the three remaining districts. This approach also works at 8 districts, where the unilateral redistrictor can win 7 of 8 seats.

If a state had an evenly split vote and no contiguity constraint, then a redistrictor could pack opposing party voters into one district and win all remaining districts no matter the number of districts. However, because districts must be contiguous, geography begins to limit how efficiently voters may be packed and cracked. As the number of districts increases, this problem increasingly limits the level of gerrymandering. In our example, once the unilateral redistrictor must draw 200 districts, he or she is able to edge out only a 0.2 definer’s advantage. As the number of districts converges to the number of individuals in the state, the advantage from controlling redistricting will diminish to zero.

Second, why does DCP begin to exhibit a second-mover/combiner’s advantage once the number of districts surpasses some threshold? Note first that no matter the number of districts the combine step prevents the definer from fully taking advantage of the ability to pack and crack. Attempts to do so can be mitigated through the combine step. However, once the definer must draw over some threshold number of districts, then the second-mover/combiner has so many possible options (on average) that he or she can wrest some of the ability to pack and crack back from the definer. To take this to an extreme, consider a case where the definer is drawing as many sub-districts as there are people in a state. In this case, the second-mover/combiner faces a problem equivalent to unilateral redistricting (i.e., taking the full population $N$ and creating $\frac{N}{2}$ districts); the combiner therefore retains advantages similar to those of a unilateral redistrictor outlined earlier in this section.

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40 The proof: Suppose each party comprised half of each district (and breaking ties randomly would lead to an expected number of wins for each party equal to half the number of districts). Then any change in the balance of voters between districts will create a majority for each party in at least one other district. Hence no other configuration of voters can be reached without each party winning at least one district.
F. Partisan Clustering Calculations

We use Moran’s I as a measure of the degree of geographic clustering among partisans, both in our simulated grid example and with the precinct-level election data for each of the states in our analysis. We use the following formula to calculate Moran’s I:

$$I = \frac{N}{W} \sum_i \sum_j w_{ij} (v_i - V)(v_j - V) / \sum_i (v_i - V)$$

Where $N$ is the number of spatial units, $v_i$ and $v_j$ are the vote shares of grid square $i$ and $j$ respectively, $V$ is the average of the vote share across the entire simple grid, $w_{ij}$ are spatial weights, and $W$ is equal to the sum of the weights $\sum_{ij} w_{ij}$. In the simple grid analysis presented in SI Appendix E.4, the vote shares $v_i$ and $v_j$ will either be 0 or 1 and we use “neighbor” weights such that $w_{ij} = 1$ if grid squares $i$ and $j$ are adjacent rook neighbors and 0 otherwise.

Figure S10 shows examples of different configurations of voters along with the corresponding Moran’s I measure for each, holding the overall map vote share for each party at a constant 0.5. Figure S10 (a) displays a high clustering scenario, where all voters from each party are packed on either side of the grid, with a Moran’s I of 0.796. Figure S10 (b) shows the case where voters from each party are perfectly evenly spread out across the grid, resulting in a Moran’s I of −1. Figure S10 (c) shows a case where Moran’s I is approximately 0, indicating neither clustering nor a tendency towards even dispersion. And Figure S10 (d) shows an example of a “city” of Party A voters (white dots) surrounded by Party B voters (black dots), and demonstrates that in this sort of geographic setup Moran’s I is 0.479, indicating significant geographic clustering.

Figure S11 (a) and (b) demonstrate scenarios where the overall vote share for Party A (white dots) of the map is much lower than $V = 0.5$, but the geographic clustering of Party A voters remains high. Figure S11 (c) and (d) show random draws from our simulation procedure, this time again with the vote shares of each party set to 0.5, but with only a slightly positive (c) or slightly negative (d) Moran’s I clustering measure.

The range of Moran’s I in the clustering results presented in SI Appendix E.4 with our simple grid simulations are limited due to the fact that our random sampling procedure for the partisanship of each voter makes it highly unlikely that very negative values of Moran’s I will occur naturally. This is because, in general, there are many different possible ways to cluster voters — in one corner of the map, in another corner, in the middle, etc. — but only a small number of ways to have voters evenly dispersed. For example, in order to have a Moran’s I of −1, voters of each partisan affiliation need to be evenly spread across the entire map (see Figure S10 (b)). Given a probability of 50% that a voter will be for Party A or Party B, this means that the probability of Moran’s I being exactly −1 is equal to $2 \times (0.5^{50})$, which is approximately 1-in-500 million.41 This is not a significant limitation of our clustering analysis because a map with clustering close to −1 become impossible to gerrymander and, when looking at real precinct-level map data, all states we use in our paper demonstrate significant geographic clustering of partisans.

Figure S12 plots the relationship between seats won by Party A and Moran’s I (as shown in Figure S7), but with each plot depicting results for a different overall Party A vote share, ranging from 40% to 60%. In general, the pattern is similar to the case where statewide vote share is split 50-50; in all cases, $\delta_D^{U}$ is substantially lower than $\delta_D^{U}$, indicating a significant reduction in the advantage conferred to the party controlling the redistricting process and in bias due to redistricting. Thus, even if vote shares of each party vary in addition to level of geographic clustering of political parties, DCP still proves effective.

41 There are two possible ways to do have perfect dispersion - one starting with a black dot in the corner and alternating across the rest of the grid, and one starting with a white dot.
Fig. S10. Ranges of Moran’s I with Different Vote Distributions
Fig. S11. Samples of Moran's I with Different Vote Distributions
Fig. S12. Define-Combine Results, by Vote Share and Moran’s I, 40%-60%
### G. Gerrychain

Gerrychain builds ensembles of districting plans using Markov chain Monte Carlo. It is developed by the [Metric Geometry and Gerrymandering Group](https://www.mggg.org). We use Gerrychain to generate our simulated district maps.

Maps are drawn and evaluated using precinct-level population data and election results. In each chain we restricted population deviations to 2% for Congressional districts and 10% for State Senate districts, and used a different set of initial districts to increase the probability of simulating districts across the full possible distribution. Full replication code available upon publication. We limited the number of cut edges in our simulations to require a moderate degree of compactness in each map, without overly constraining the set of maps.

For each state, we used the Gerrychain algorithm MGGG (2018) to generate 10 independent Markov chains of 100,000 maps of $2N$ subdistricts, where $N$ is the number of districts in that state, resulting in 1,000,000 define-stage maps per state.\(^{42}\) We thinned the chain by selecting 1% of the simulated maps and then generated up to 500 possible combinations of pairs of districts to evaluate. For some of the maps, particularly in states with fewer Congressional districts, there were not 500 possible combinations of districts for every defined map. In these cases we used all possible unique combinations.

In a small number of cases, there are no valid combinations for a generated map. Figure S13 illustrates one case of this in Massachusetts. Overall, among the 1,000,000 maps generated across ten states, only 404 (0.04%) of the generated maps did not have a single valid combination. In states with a small number of districts, it is possible to draw define-stage maps that allow only one valid combination. This is a particular issue in states like Iowa, where only 8 subdistricts are drawn for the Congressional district map. In our simulations, we restricted our analysis only to define-stage maps that allow at least 10 combinations. However, this is not a significant constraint for any map except Iowa’s Congressional districts, and does not change the ultimate result in any other state. We also investigated implementing other constraints, such as a limit on the number of subdistricts that could only have one neighbor. However, we found that restricting this did not affect the ultimate result, and therefore was unnecessary.

Tables S2 and S3 summarize the number of maps drawn and sets generated for each state and district type. Using actual election results, we then identified the best map for the Democrats and for the Republicans under URP and under DCP.

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\(^{42}\)Maps are drawn and evaluated using precinct-level population data and election results. In each chain we restricted population deviations to 2% for Congressional Districts and 10% for State Senate Districts, and used a different set of initial districts to increase the probability of simulating districts across the full possible distribution. Full replication code available upon publication.
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Table S3. Gerrychain Result Summary for State Senate Maps

Fig. S13. Example of a simulated map for Massachusetts with no valid combinations. Districts 8 and 16 in the southeastern portion of the state are both only adjacent to District 9. If Districts 8 and 9 are paired, there are no contiguous districts with which to pair District 16. Similarly, if Districts 16 and 9 are paired, there are no contiguous districts with which to pair District 8. Therefore the combine stage is impossible for this map.
Fig. S14. Georgia 2016 Presidential Election Results, Precinct Level. This map shows the high extent of partisan clustering in Georgia. The spatial distribution of voters constrains the extent which districts can be drawn without partisan bias, even when there is not explicit intent towards partisan gerrymandering.
H. State Simulation Results with Additional Elections

We present the main results of our state-by-state simulations in Figure 3—here without uniform swing and using a range of different past elections. Figures S15–S25 present additional results for presidential and gubernatorial elections in each state. The set of elections varies based on the data available in each state.

Georgia

Congressional Districts

2016 President
(47% Dem.)

Democratic Seats

☐ D Alone  ○ D then R  ● R then D  ■ R Alone

Fig. S15. Gerrychain Results: Georgia

Iowa

Congressional Districts

2016 President
(45% Dem.)

Democratic Seats

☐ D Alone  ○ D then R  ● R then D  ■ R Alone

Fig. S16. Gerrychain Results: Iowa
Maryland

2012 President (61% Dem.)

2014 Governor (46% Dem.)

2016 President (58% Dem.)

2018 Governor (42% Dem.)

Democratic Seats

Fig. S17. Gerrychain Results: Maryland
Massachusetts

Congressional Districts

State Senate Districts

2012 President (62% Dem.)

2014 Governor (49% Dem.)

2016 President (65% Dem.)

2018 Governor (33% Dem.)

Democratic Seats

☐ D Alone  ○ D then R  ● R then D  ■ R Alone

Fig. S18. Gerrychain Results: Massachusetts

Michigan

Congressional Districts

State Senate Districts

2016 President (49% Dem.)

Democratic Seats

☐ D Alone  ○ D then R  ● R then D  ■ R Alone

Fig. S19. Gerrychain Results: Michigan
Minnesota

Congressional Districts

State Senate Districts

2012 President (54% Dem.)

Democratic Seats

Fig. S20. Gerrychain Results: Minnesota

North Carolina

Congressional Districts

State Senate Districts

2012 President (49% Dem.)

Democratic Seats

Fig. S21. Gerrychain Results: North Carolina
Oregon

Congressional Districts

State Senate Districts

2016 President
(56% Dem.)

Democratic Seats

D Alone  D then R  R then D  R Alone

Fig. S22. Gerrychain Results: Oregon

Pennsylvania

Congressional Districts

State Senate Districts

2012 President
(53% Dem.)

2014 Governor
(55% Dem.)

2016 President
(50% Dem.)

Democratic Seats

D Alone  D then R  R then D  R Alone

Fig. S23. Gerrychain Results: Pennsylvania
Texas

Congressional Districts

2012 President (42% Dem.)

2014 Governor (40% Dem.)

2016 President (45% Dem.)

State Senate Districts

Democratic Seats

D Alone  D then R  R then D  R Alone

Fig. S24. Gerrychain Results: Texas

Virginia

Congressional Districts

2016 President (53% Dem.)

2017 Governor (54% Dem.)

State Senate Districts

Democratic Seats

D Alone  D then R  R then D  R Alone

Fig. S25. Gerrychain Results: Virginia
References


Duchin, Moon. 2018. “Gerrymandering Metrics: How to Measure? What’s the Baseline?”


